ENDOGENOUS LOBBYING

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Abstract
In this article, we present a citizen-candidate model of representative democracy with endogenous lobbying. We find that lobbying induces policy compromise and always affects equilibrium policy outcomes. In particular, even though the policy preferences of lobbies are relatively extreme, lobbying biases the outcome of the political process toward the center of the policy space, and extreme policies cannot emerge in equilibrium. Moreover, in equilibrium, not all lobbies participate in the policy-making process. (JEL: D72, D74, D78)

1. Introduction

A long tradition in political economy builds on the assumption that the main objective of politicians is to win an election (Downs 1957). Within this framework, known as the "Downsian" paradigm, (office-concerned) political candidates shape their policy platforms to please the (policy-concerned) electorate so as to maximize their probability of winning. In other words, a building block of the Downsian paradigm is that the preferences of political candidates differ from the preferences of the citizens, or equivalently, the (prespecified) set of political candidates is not a subset of the citizenry.

Several authors have challenged this view by proposing alternative models of electoral competition where politicians are assumed to be not only office-motivated, but also policy-motivated (Alesina 1988; Hibbs 1977; Wittman 1977). Within this framework, known as the "partisan" paradigm, political candidates choose their policy platforms by trading off their policy concerns with their desire

to win the election. As in the Downsian framework, however, the set of political candidates is exogenous.

Recently, Besley and Coate (1997) and Osborne and Slivinski (1996) have proposed an alternative approach to the study of political competition known as the "citizen-candidate" paradigm. This framework removes the artificial distinction between citizens and candidates prevalent in the other approaches. This is accomplished by assuming that politicians are selected by the people from those citizens who choose to become candidates in an election. Once in office, elected candidates implement their most preferred policies.

While ultimately implemented by elected representatives, policies are typically the outcome of a political process that also involves nonelected political actors. In particular, lobbying is an important part of the policy-making process in representative democracies. This raises the question: To what extent does lobbying affect policy?

Several authors have addressed this issue in the context of models of electoral competition where lobbies (or interest groups) compete to influence policy-makers.[1] In most of the recent literature, lobbying is modeled as a "menu-auction," where exogenously given lobby groups offer policy-makers contribution schedules, representing binding promises of payment, conditional on the chosen policy (Bernheim and Whinston 1986; Besley and Coate 2001; Dixit, Grossman, and Helpman 1997; Grossman and Helpman 1994,1996; Persson and Helpman 1998). [2]

An implicit assumption of the menu-auction model of lobbying is that all lobbies participate in the policy-making process. We find this assumption problematic for at least two reasons. First, casual observations suggest that although a number of lobby groups may be willing to offer favors to elected politicians in exchange for policy compromise, policy-makers have a choice as to whom to accept favors from. Second, empirical evidence suggests that many existing lobby groups are often dormant and neither raise nor spend any money (Wright 1996).

In this article, we propose an alternative model of lobbying where the elected policy-maker chooses the lobbies that participate in the policy-making process. In our framework, policy is the outcome of efficient bargaining between the elected

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1.   This literature originates from the work by Tullock (1967) on rent-seeking. For a partial account of the large literature on lobbying see, for example, Grossman and Helpman (2001) or chapter 7 in Persson and Tabellini (2000) and the references therein. A substantial part of the literature has focused on the incentives for lobbies to gather information and provide it to the policy-makers (e.g., Austen-Smith and Wright 1992; Grossman and Helpman 2001). Like Besley and Coate (2001), Grossman and Helpman (1996), Persson and Helpman (1998), and many others, we abstract from the informational role of lobbies and focus instead on their influence-seeking activities.

2.   In some models, payments take the form of campaign contributions (e.g., Grossman and Helpman 1996). In other models, they take the form of lobbying expenditures that provide post-election support to officeholders (e.g., Besley and Coate 2001; Persson and Helpman 1998).

policy-maker and a coalition of lobbies selected by the policy-maker.[3] This is the sense in which lobbying is endogenous in our model.

We consider a citizen-candidate model of electoral competition that builds on the work by Besley and Coate (1997, 2001). As in Besley and Coate (1997), we model the political process as a multistage game that begins with the citizens' decisions to participate in the political process as candidates for public office. Given the set of candidates, citizens vote in an election that selects the plurality winner to choose policy for one period. When casting their ballot, citizens are assumed to be strategic.[4]

As in Besley and Coate (2001), we assume that after the election lobbies try to influence the policy choice of the elected candidate by offering him transfers in exchange for policy compromise. Contrary to Besley and Coate (2001), however, we do not model lobbying as a menu-auction, where all lobbies are (exogenously) assumed to participate in the policy-making process.[5] Rather, we assume that given the set of existing lobbies, the elected candidate (endogenously) chooses the coalition of lobbies he will bargain with over policy in exchange for transfers.

Our main results can be summarized as follows. First, lobbying induces policy compromise. The equilibrium policy outcome is always a compromise between the policy preferences of the elected candidate and the policy preferences of the lobbies that participate in the policy-making process. The extent of the compromise depends on the relative intensity of the policy motivation of the elected candidate *vis-a-vis* the lobbies. We believe that compromise is a natural consequence of lobbying and is also an implication of the menu-auction model of lobbying (Besley and Coate 2001; Grossman and Helpman 1996).[6]

Second, not all lobbies participate in the policy-making process. In equilibrium, no elected candidate ever includes all lobbies in the bargaining process that determines the policy outcome. This result is consistent with the empirical evidence cited above. Moreover, it highlights the fact that assuming that all lobbies participate in the decision-making process is not without consequences and sets our framework apart from the menu-auction approach.

Third, lobbying matters. In our model, even though the policy preferences of all potential candidates span the entire policy space, the lobbying process reduces the set of policies that can be implemented in equilibrium. This result

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3.   Diermeier and Merlo (2000) use a similar framework to analyze the process of government formation in parliamentary democracies.

4.   This assumption differentiates the citizen-candidate model of Besley and Coate (1997) from the one of Osborne and Slivinski (1996) where citizens are assumed to vote sincerely. In Section 5.2 we consider a version of our model with sincere voting.

5.   In the remainder of the article, we refer to this approach as exogenous lobbying.

6.   Notice, however, that Grossman and Helpman (1996) consider a Downsian model of electoral competition where candidates choose policies to maximize their probability of winning. In their model, lobbying induces candidates to adopt policies that represent a compromise between the policy preferences of lobbies and those of voters.

is in contrast with the findings of Besley and Coate (2001). In their model of exogenous lobbying, the presence of lobbies in the political process need have little or no effect on equilibrium policy outcomes. In particular, they show that it is possible to construct examples where the equilibrium sets of policy outcomes of the games with and without lobbying coincide with the set of feasible policies. The reason for the result is that voters can restrict the influence of lobbies via strategic delegation by supporting candidates with offsetting policy preferences. In other words, in the game where lobbies are allowed to influence policy, voters can strategically elect a candidate who (after lobbying takes place) implements exactly the same policy that a different candidate would implement in the game where lobbying is ruled out. The feature of their model that is critical to obtain this result is the freedom to choose the characteristics of the lobbies that participate in the policy-making process (i.e., the menu-auction game). In our model, lobbying takes the form of bargaining between the elected candidate and a coalition of lobbies of his choice. In equilibrium, not all lobbies are selected to participate in the policy-making process for any elected candidate, and not all feasible policies can be implemented.

Fourth, lobbying biases the outcome of the policy-making process toward the center of the policy space. In our model, even though the policy preferences of lobbies are relatively extreme, lobbying has a moderating effect on policy, and extreme policies never emerge as an equilibrium outcome of the political process. This result is at odd with the findings of other existing models where lobbying tends to induce policy outcomes that are relatively extreme (Austen-Smith 1987; Baron 1994; Groseclose and Snyder 1996; Grossman and Helpman 1996).[7] The intuition for this result is as follows. In equilibrium, the candidates who run for office are citizens with relatively extreme policy preferences. If elected, they include in their bargaining coalition lobbies whose policy preferences are on the opposite end of the policy spectrum than their own preferences. This maximizes the transfers they receive for compromising on their policy choices. The outcomes of the compromise are policies that are relatively moderate (that is, policies that are near the center of the policy space). This implication of our model is consistent with the empirical evidence presented by Austen-Smith and Wright (1994), which shows that special interest groups often lobby legislators whose policy positions, prior to any lobbying, are diametrically opposed to theirs.

Before turning our attention to the model, we would like to stress that in this article we focus on only one of the many activities interest groups typically engage in: lobbying elected politicians. In particular, our analysis abstracts from the fact that interest groups often spend a considerable amount of resources in campaign contributions, trying to get candidates whose policy platforms they like elected.

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7.   Notice that this literature treats candidate entry as exogenous and hence ignores the effects of lobbying on the type of citizens who run for public office.

Rather, we emphasize the fact that many interest groups also spend a considerable amount of resources after elections trying to persuade already elected politicians whose policy preferences they do not like to change their mind.

The rest of the article is organized as follows. In Section 2, we present the model. Section 3 contains the characterization of the equilibria and Section 4 a discussion of the results. In Section 5, we consider several extensions of the model and assess the robustness of our results. We conclude with Section 6.

2. The Model

Each citizen $i \in \{1, \ldots, N\}$ has quasi-linear preferences over a one-dimensional policy outcome $x \in X = [-1, 1]$ that has a public good nature and distributive benefits $y_i \in \mathbb{R}$ that have a private good nature. Citizens differ with respect to their policy preferences. We assume there exists a continuum of types of citizens indexed by $j \in X$, where j denotes the most preferred policy outcome of all citizens of that type.[8] Let F denote the cumulative distribution function of citizens' types over the support X. We take the density of F to be continuous and symmetric around its median 0. We assume that the number of citizens N is large. Moreover, to guarantee that every $j \in X$ is represented in the citizenry, we abuse notation and refer to the population of citizens as a unit mass with density f on X.

The utility function of citizen i of type j (henceforth, citizen i^j) is given by

$$U(x, y_i, j) = u(x, j) + \lambda y_i, \tag{1}$$

where $u(x, j)$ is strictly concave in x, single-peaked and symmetric around j, and $\lambda > 0$ measures the intensity of each citizen's preferences over money with respect to policy.[9]

As discussed in the Introduction, we model policy-making as the outcome of a political process that involves not only the citizen who is elected by the citizenry to represent them, but also nonelected political agents known as lobbies. We assume there is a finite number of lobbies H that differ with respect to their policy preferences. Each lobby $h \in \{1, \ldots, H\}$ has a most preferred policy outcome $l_h \in X$ and preferences represented by

$$V(x, y_h, l_h) = v(x, l_h) + \mu y_h, \tag{2}$$

where $v(x, l_h)$ is strictly concave in x, single-peaked and symmetric around l_h, and $\mu > 0$ measures the intensity of each lobby's preferences over money with

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8. As in Besley and Coate (1997) there is no incomplete information in our model. In particular, the type of each citizen is publicly observable.

9. Notice that if $\lambda = 0$ citizens are purely policy-motivated and lobbying is irrelevant. This is the case studied in Besley and Coate (1997). We therefore restrict attention to the case where λ is strictly positive.

respect to policy. To capture the idea that lobbies care relatively more about money than citizens we assume that $\mu \geq \lambda$.[10]

For ease of exposition—in order to obtain closed-form solutions to the model—in what follows we take[11]

$$u(x, j) = -(x - j)^2 \tag{3}$$

and

$$v(x, l_h) = -(x - l_h)^2. \tag{4}$$

We normalize aggregate transfers to be zero (i.e., $\sum_{i \leq N} y_i + \sum_{h \leq H} y_h = 0$). Also, we assume that any policy $x \in X$ is costless to implement. Furthermore, we restrict attention to the case where there are three lobbies labelled L, C, and R, with most preferred policy outcomes $l_L = -1$, $l_C = 0$, and $l_R = 1$, respectively. Notice that l_L, l_C, and l_R denote, respectively, the left, center, and right of the policy space X, and l_C is the median of the distribution of the citizens' types. In the remainder of the article, we denote $\mathcal{L} = \{L, C, R\}$ the set of lobbies.

We typically think of lobbies as representing a wide range of policy preferences. In particular, while some lobbies may hold rather extreme views on either side of the political spectrum, other lobbies may hold more moderate views. Our specification of the set of lobbies $\mathcal{L} = \{L, C, R\}$ is the simplest one that captures this insight. However, our setup can be extended to include any finite number of lobbies.

We assume that the political process has three stages. In the first stage, all citizens choose whether to run for office. Given the set of candidates that have entered the electoral competition an election follows in the second stage. The election selects one candidate that is delegated the policy decision for one period. In the third and final stage, lobbying takes place and policy is chosen. We describe below the structure of each stage of the political process.

2.1. Entry of Candidates

Each citizen must decide simultaneously and independently whether or not to run for office. If a citizen enters the electoral competition as a candidate he has to pay a (small) monetary cost $\delta > 0$. The decision to run for office yields benefits to the citizen either directly by winning or indirectly by affecting the identity of the winner.

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10. This assumption seems natural in light of the fact that lobbies are typically corporations. However, our analysis can be easily extended to the case where $\mu < \lambda$ without changing the main thrust of our results.

11. Although the details of the derivation presented in the article clearly depend on the quadratic form of the functions $u(\cdot, \cdot)$ and $v(\cdot, \cdot)$, all of our results hold true for all strictly concave, single-peaked, and symmetric functions.

Let $\sigma(i^j) \in \{0, 1\}$ denote the decision by citizen i^j whether to become a candidate: $\sigma(i^j) = 1$ indicates citizen i^j's decision to enter the electoral competition.[12] Let $\sigma = (\sigma(1), \ldots, \sigma(N))$ denote the vector of all citizens' entry decisions. For any given σ let $\mathcal{C}(\sigma) = \{i^j | \sigma(i^j) = 1\}$ denote the set of candidates with typical element e. This set is the outcome of the entry-of-candidates subgame.

In the event that no citizen runs for office, we assume that a default policy $x_0 \in X$ is implemented.

2.2. Voting

Elections are structured so that each citizen has one vote that, if used, must be cast for one of the candidates.

In particular, given a set of candidates $\mathcal{C}(\sigma)$, each citizen simultaneously and independently decides to vote for any candidate in $\mathcal{C}(\sigma)$ or abstains. Let $\gamma(i^j)$ denote citizen i^j's choice: if $\gamma(i^j) = e$ then citizen i^j casts a vote for candidate $e \in \mathcal{C}(\sigma)$; if $\gamma(i^j) = 0$, he abstains. The vector of all citizens' voting decisions is denoted by $\gamma = (\gamma(1), \ldots, \gamma(N))$.

The candidate who receives the most votes is elected, and in the event of ties, the winning candidate is chosen with equal probability from among the tying candidates.[13] We denote $P^E \in \mathcal{C}(\sigma)$ the elected candidate, where $E \in X$ denotes the elected candidate's most preferred policy outcome.

We assume that citizens correctly anticipate the outcome of the lobbying stage that follows an election and vote strategically: Each citizen i^j makes his voting decision $\gamma(i^j)$ so as to maximize his expected utility given the decisions of all other citizens.[14]

2.3. Lobbying

Each lobby $h \in \mathcal{L}$ is assumed to be able to sign binding contracts on policy choices with the elected candidate P^E in exchange for transfers. Notice that the elected candidate P^E has the option of not signing any contract and implement his most preferred policy E.[15]

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12. In principle, we could allow candidates to randomize on their entry decision. However, as in Besley and Coate (1997), we restrict attention to equilibria in pure strategies.

13. Notice that although it is critical for our analysis that in case of a tie all tying candidates have a strictly positive probability of winning, the assumption that these probabilities are equal is of little consequence.

14. In Section 5.2 we consider an alternative specification of our model where citizens vote sincerely.

15. If the elected candidate chooses this option, then the model coincides with the original model of Besley and Coate (1997), where lobbying is not allowed.

Let

$$\Delta = \{\{\emptyset\}, \{L\}, \{C\}, \{R\}, \{L, C\}, \{C, R\}, \{L, R\}, \{L, C, R\}\}$$

denote the power set of $\mathcal{L}$ with typical element ℓ. The set Δ is the collection of all possible coalitions of lobbies with whom the elected candidate P^E may choose to bargain over policy and transfers.

We model lobbying as a two-stage bargaining game. In the first stage, each possible coalition $\ell \in \Delta$ is associated with a willingness to pay, $W_\ell(x, E)$, for any policy $x \in X$ the elected candidate P^E may choose to implement instead of his most preferred policy E:

$$W_\ell(x, E) = \sum_{h \in \ell} w_h(x, E), \tag{5}$$

where $w_h(x, E)$ is the willingness to pay of lobby h measured in units of the private good and $W_\emptyset(x, E) \equiv 0$.

Given the preferences of a lobby specified in equation (2), the willingness to pay of lobby $h \in \mathcal{L}$ for any policy $x \in X$ implemented by the elected candidate P^E is

$$w_h(x, E) = \frac{v(x, l_h) - v(E, l_h)}{\mu}. \tag{6}$$

This is the monetary value of the utility gain (or loss) with respect to the status quo that lobby h obtains if the elected candidate P^E's policy choice is x. The status quo is here defined to be P^E's policy choice in the absence of any lobbying, E.[16]

From (5) and (6) we obtain the total willingness to pay of coalition $\ell \in \Delta$ for a given policy choice $x \in X$ by the elected candidate P^E:

$$W_\ell(x, E) = \sum_{h \in \ell} \frac{v(x, l_h) - v(E, l_h)}{\mu}. \tag{7}$$

In the second stage of the bargaining game, the elected candidate P^E first chooses an optimal policy $x_{PE}(\ell)$ for any potential coalition $\ell \in \Delta$:

$$x_{PE}(\ell) \in \underset{x \in X}{\operatorname{argmax}}\, u(x, E) + \lambda\, W_\ell(x, E), \tag{8}$$

and then chooses a bargaining coalition ℓ_{PE}:

$$\ell_{PE} \in \underset{\ell \in \Delta}{\operatorname{argmax}}\, u(x_{PE}(\ell), E) + \lambda\, W_\ell(x_{PE}(\ell), E). \tag{9}$$

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16.   A direct implication of (6) is that for any policy $x \in X$, the willingness to pay of a lobby with the same most preferred policy as the elected candidate is nonpositive.

Hence, an outcome of the bargaining game between the elected candidate P^E and a selected coalition ℓ_{pE} is a policy choice $x_{pE}(\ell_{pE})$ and transfers $W_{\ell_{pE}}(x_{pE}(\ell_{pE}), E)$.

Implicit in the statement of problems (8) and (9) is the assumption that the elected candidate appropriates the entire willingness to pay of the selected bargaining coalition. This is equivalent to assuming that at the lobbying stage the elected candidate has all the bargaining power.[17]

3. Results

We proceed backward to solve for the *subgame perfect equilibria* of the three-stage political game described in Section 2 above. We start from the last stage of the game: lobbying.

3.1. Equilibria of the Lobbying Subgame

Let P^E be the candidate elected in the voting subgame. We begin our analysis by characterizing the elected candidate P^E's optimal coalition choice $\ell_{pE} \in \Delta$ and optimal policy choice $x_{pE} \in X$.

We first show that for any coalition $\ell \in \Delta$ the equilibrium policy choice that the lobbying process generates is uniquely determined.[18]

LEMMA 1.   *For any elected candidate $P^E \in \mathcal{C}(\sigma)$ and any coalition $\ell \in \Delta$, there exists a unique optimal policy choice $x_{pE}(\ell)$ that solves problem (8):*

$$x_{pE}(\ell) = \frac{1}{1 + \rho\,|\ell|}\left(E + \rho\sum_{h\in\ell}l_h\right), \tag{10}$$

where

$$\rho \equiv \frac{\lambda}{\mu}. \tag{11}$$

The proof of Lemma 1 is presented in the Appendix.

The outcome of the bargaining is a compromise between the policy preferences of the elected candidate and those of the lobbies included in the bargaining

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17.   This assumption is not critical for our results. The equilibrium characterization of the lobbying subgame remains the same (up to the size of the transfers) if the gains from trade are shared between the elected candidate and the members of the coalition in any fixed proportion.

18.   This result is similar to the one obtained by Diermeier and Merlo (2000) in the context of government coalition bargaining.

coalition. Given the quadratic specification of preferences we adopt, this policy compromise takes the form of a weighted average of the most preferred policies of the parties involved in the negotiation, because, by assumption, $\rho \leq 1$, the elected candidate's policy preferences are weighted more favorably than the policy preferences of the lobbies included in the bargaining coalition. This implies that the stronger the policy motivation of the elected candidate relative to that of the lobbies, the closer the equilibrium policy is to the one most preferred by the elected candidate.

We can now complete our characterization of the lobbying stage of the model by analyzing the elected candidate P^E's choice of the optimal lobbying coalition $\ell_{pE} \in \Delta$.

LEMMA 2.   *For any elected candidate $P^E \in \mathcal{C}(\sigma)$ the optimal coalition choice $\ell_{pE} \in \Delta$ that solves problem (9) is,*

If $-1 \leq E \leq -\tau(\rho)$, then $\ell_{pE} = \{C, R\}$;
if $-\tau(\rho) \leq E \leq 0$, then $\ell_{pE} = \{R\}$;
if $0 \leq E \leq \tau(\rho)$, then $\ell_{pE} = \{L\}$;
if $\tau(\rho) \leq E \leq 1$, then $\ell_{pE} = \{L, C\}$;

where

$$\tau(\rho) \equiv \frac{\sqrt{2\rho^2 + 3\rho + 1} - 1}{2\rho + 3}. \tag{12}$$

The proof of Lemma 2 is presented in the Appendix.

An immediate consequence of Lemma 2 is that no elected candidate ever chooses to implement his most preferred policy. Thus, lobbying always occurs in equilibrium and influences the policy choice of any elected candidate.

Another consequence of Lemma 2 is that in all equilibria no candidate ever includes all lobbies in his bargaining coalition. In equilibrium, there always exists at least one lobby that is excluded from the policy-making process and does not make any transfer to the elected candidate. Which lobbies are excluded depends on the policy preferences of the elected candidate.

We have now all the elements to present our first result. This result summarizes the outcome of the lobbying subgame for any possible elected candidate P^E.

PROPOSITION 1.   *For any elected candidate $P^E \in \mathcal{C}(\sigma)$ the optimal coalition choice $\ell_{pE} \in \Delta$, policy choice $x_{pE} \in X$ and transfers $W_{\ell_{pE}}$ are*

If $-1 \leq E \leq -\tau(\rho)$, then

$$\ell_{pE} = \{C, R\}, \quad x_{pE} = \frac{E + \rho}{1 + 2\rho}, \quad W_{\ell_{pE}} = \frac{(2E - 1)^2 (1 + \rho) 2\rho}{\mu (1 + 2\rho)^2}.$$

If $-\tau(\rho) \leq E \leq 0$, *then*

$$\ell_{PE} = \{R\}, \quad x_{PE} = \frac{E + \rho}{1 + \rho}, \quad W_{\ell_{PE}} = \frac{(E - 1)^2 (2 + \rho) \rho}{\mu (1 + \rho)^2};$$

If $0 \leq E \leq \tau(\rho)$, *then*

$$\ell_{PE} = \{L\}, \quad x_{PE} = \frac{E - \rho}{1 + \rho}, \quad W_{\ell_{PE}} = \frac{(E + 1)^2 (2 + \rho) \rho}{\mu (1 + \rho)^2};$$

If $\tau(\rho) \leq E \leq 1$, *then*

$$\ell_{PE} = \{L, C\}, \quad x_{PE} = \frac{E - \rho}{1 + 2\rho}, \quad W_{\ell_{PE}} = \frac{(2E + 1)^2 (1 + \rho) 2\rho}{\mu (1 + 2\rho)^2}.$$

Proof. The proof follows directly from Lemma 1, Lemma 2, and equation (7). $\qquad\square$

As shown in Proposition 1, the equilibrium of the lobbying subgame is such that the elected candidate P^E receives strictly positive transfers $W_{\ell_{PE}}$ from coalition ℓ_{PE} for implementing policy x_{PE}. It is easy to show that the more lobbies value money over policy (that is, the higher is μ) and the less the elected candidate values money over policy (that is, the lower is λ), the smaller are these transfers.

Let $\bar{\rho} \in (0, 1)$ be implicitly defined by the following equation:

$$(1 + 2\bar{\rho}) \tau(\bar{\rho}) = 3\bar{\rho}^2 + \bar{\rho} - 1. \tag{13}$$

where $\tau(\cdot)$ is defined in (12).

COROLLARY 1.   *The lobbying process implies that, for every $\rho > 0$, not all policy choices $x \in X$ can be implemented in equilibrium. In particular, if $\rho \leq \bar{\rho}$ the set of policy choices $X_1 \subset X$, $X_1 \neq \emptyset$, defined by*

$$X_1 = \left[-1, \min \left\{ -\frac{1 - \rho}{1 + 2\rho}, -\frac{\rho}{1 + \rho} \right\} \right] \cup \left[\max \left\{ \frac{1 - \rho}{1 + 2\rho}, \frac{\rho}{1 + \rho} \right\}, 1 \right], \tag{14}$$

cannot be implemented in equilibrium.

If instead $\rho \geq \bar{\rho}$ the set of policy choices $X_2 \subset X$, $X_2 \neq \emptyset$, defined by

$$X_2 = \left[-1, -\frac{\rho}{1 + \rho} \right] \cup \left[-\frac{\rho - \tau(\rho)}{1 + \rho}, \min \left\{ -\frac{1 - \rho}{1 + 2\rho}, -\frac{\rho - \tau(\rho)}{1 + 2\rho} \right\} \right]$$

$$\cup \left[\max \left\{ \frac{1 - \rho}{1 + 2\rho}, \frac{\rho - \tau(\rho)}{1 + 2\rho} \right\}, \frac{\rho - \tau(\rho)}{1 + \rho} \right] \cup \left[\frac{\rho}{1 + \rho}, 1 \right], \tag{15}$$

cannot be implemented in equilibrium.

Proof. The result follows from Proposition 1.   □

Corollary 1 shows that even though the set of potential candidates spans the entire policy space, the lobbying process reduces the set of policies that are implementable. Hence, we conclude that lobbying matters: Lobbying changes the set of implementable policy outcomes. In particular, lobbying prevents the political process from implementing policies that are relatively extreme.

We can now turn our attention to the analysis of the voting stage of the model.

3.2. Equilibria of the Voting Subgame

We begin our analysis by restricting attention to the set of two-candidate equilibria of the electoral model. This implies that when analyzing the voting subgame we focus on voting when two or at most three candidates enter the electoral competition.[19] In Section 5.1 we extend our analysis to the case of multiple candidates and show that our results also hold in this more general case. Our analysis of the voting subgame parallels the analysis of Besley and Coate (1997). In particular, we rule out weakly dominated voting strategies.

A voting strategy $\gamma(i^j)$ is weakly dominated for citizen i^j if there exists an alternative voting strategy $\hat{\gamma}(i^j)$ for i^j such that, for every configuration of the voting profile of the other citizens, citizen i^j's payoff associated with $\gamma(i^j)$ is less than or equal to the payoff associated with $\hat{\gamma}(i^j)$.

Restricting attention to equilibria that survive one round of elimination of weakly dominated voting strategies greatly simplifies the analysis of the voting subgame when there is more than one candidate. In particular, we can prove the following proposition.

PROPOSITION 2.   *Assume that $\mathcal{C}(\sigma)$ contains at least two candidates who, if elected, implement different policy choices. All equilibria of the voting subgame that survive one round of elimination of weakly dominated strategies are such that no citizen i^j ever votes for any candidate $\underline{e} \in \mathcal{C}(\sigma)$ that, if elected, implements i^j's least preferred policy within the set of equilibrium policy choices of the candidates in $\mathcal{C}(\sigma)$*

$$\underline{e} \in \operatorname*{argmax}_{e \in \mathcal{C}(\sigma)} |x_e(\ell_e) - j|. \qquad (16)$$

The proof of Proposition 2 is presented in the Appendix. This proposition states that when there are at least two candidates running for office no citizen i^j ever

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votes for his least-preferred candidate $\underline{e}$. Given our assumptions about each citizen's preferences, $\underline{e}$ is any candidate whose equilibrium policy choice is the one furthest away (among the equilibrium policies implemented by the candidates in $C(\sigma)$) from citizen i^j's most preferred policy.

Proposition 2 implies that in a two-candidate voting subgame where the two candidates implement different policies, each citizen votes for his most-preferred candidate. In other words, strategic voting coincides with sincere voting in this instance. This is not necessarily the case in a three-candidate voting subgame.

3.3. Equilibria of the Entry-of-Candidates Subgame

As indicated in Section 3.2, we focus here on the characterization of the set of two-candidate equilibria.[20] Let $C(\sigma) = \{e_1, e_2\}$ be the equilibrium set of candidates where $j_1, j_2 \in X$ denote the type of candidate e_1 and e_2, respectively.

We first show that in all two-candidate equilibria the candidates' policy choices are symmetric around the median policy 0.

LEMMA 3. *All two-candidate equilibria of the electoral competition model,* $C(\sigma) = \{e_1, e_2\}$, *are such that*

$$x_{e_1}(\ell_{e_1}) = -x_{e_2}(\ell_{e_2}). \tag{17}$$

The proof of Lemma 3 is presented in the Appendix. The intuition behind this result is that to enter the electoral competition and pay the entry cost δ each candidate must have a strictly positive probability of winning. In the contest of our model, this implies that neither candidate can win with probability 1 and both candidates must win with equal probability. Because, in two-candidate electoral competitions, citizens vote sincerely, then the population of voters necessarily has to split equally between the two candidates. This cannot occur if the distance of each candidate from the median policy differs.

There are two types of two-candidate equilibria. There are equilibria in which the two candidates are of the same type (that is, they have identical policy preferences) and equilibria in which the two candidates are of different types. We start from the latter, clearly more interesting, case.

We distinguish among three classes of two-candidate equilibria depending on whether the equilibrium policy choices exhibit *reversal*. An equilibrium policy choice exhibits reversal if it is on the opposite side of the median than the candidate's type. On the basis of this criterion we identify: *no-reversal equilibria*, where the policy choices of both candidates exhibit no reversal; *reversal equilibria*, where the policy choices of both candidates exhibit reversal; and *hybrid*

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20. Multi-candidate equilibria are discussed in Section 5.1.

equilibria, where the policy choice of one of the candidates exhibits reversal while the policy choice of the other candidate does not.

　　We start from the characterization of the set of two-candidate no-reversal equilibria.

PROPOSITION 3.   *All two-candidate no-reversal equilibria of the electoral competition model, $C(\sigma) = \{e_1, e_2\}$, where $j_1 \neq j_2$, have the following properties:*

The candidates' types are

$$j_1 \in [-1, -\rho] \qquad j_2 \in [\rho, 1] \quad and \quad j_1 = -j_2. \tag{18}$$

The equilibrium coalition choices are

$$\ell_{e_1} = \{C, R\} \qquad \ell_{e_2} = \{L, C\}. \tag{19}$$

The equilibrium policy choices are

$$x_{e_1} \in \left[-\frac{1-\rho}{1+2\rho}, 0 \right] \qquad x_{e_2} \in \left[0, \frac{1-\rho}{1+2\rho} \right], \tag{20}$$

and

$$x_{e_1} = -x_{e_2}. \tag{21}$$

　　The proof of Proposition 3 is presented in the Appendix. Intuitively, the two-candidate equilibria characterized in Proposition 3 are such that both candidates are elected with equal (and strictly positive) probability. This is enough to guarantee that each candidate wants to run for office. Indeed, in our framework, as in Besley and Coate (2001), there are rents from being the elected candidate that are generated by the lobbying process. Of course this relies on our assumption, mentioned in Section 2.1, that the cost δ of running for office is small relative to the rents that an elected candidate can capture through the lobbying process. Moreover, no other candidate is willing to enter the electoral competition provided that, in the event of a new entry, the population of voters splits so that the new entrant has zero probability of winning. Notice that in our framework it is possible to construct an off-the-equilibrium-path behavior for voters that has this feature. Indeed, following a new entry we are in a three-candidate voting sub-game and according to Proposition 2 above it is enough that voters, when they are nonpivotal, do not vote for the candidate that implements their least-preferred policy choice.

　　A key feature of the characterization of the two-candidate no-reversal equilibria presented in Proposition 3 is that the two candidates that run for office are

citizens with rather extreme policy preferences, as shown in equation (18). However, as a result of the lobbying process, they implement policies that are biased toward the center, as shown in equation (20).

Next we characterize the set of two-candidate reversal equilibria.

PROPOSITION 4.   *All two-candidate reversal equilibria of the electoral competition model, $C(\sigma) = \{e_1, e_2\}$, where $j_1 \neq j_2$, have the following properties:*

The candidates' types are

$$j_1 \in [-\rho, 0] \qquad j_2 \in [0, \rho] \quad and \quad j_1 = -j_2. \tag{22}$$

If $j_1 \in [-\rho, -\tau(\rho)]$—equivalently $j_2 \in [\tau(\rho), \rho]$—the equilibrium coalition choices are

$$\ell_{e_1} = \{C, R\} \qquad \ell_{e_2} = \{L, C\} \tag{23}$$

and the equilibrium policy choices are

$$x_{e_1} \in \left[0, \frac{\rho - \tau(\rho)}{1 + 2\rho}\right] \qquad x_{e_2} \in \left[-\frac{\rho - \tau(\rho)}{1 + 2\rho}, 0\right]. \tag{24}$$

If instead $j_1 \in [-\tau(\rho), 0]$—equivalently $j_2 \in [0, \tau(\rho)]$—the equilibrium coalition choices are

$$\ell_{e_1} = \{R\} \qquad \ell_{e_2} = \{L\} \tag{25}$$

and the equilibrium policy choices are

$$x_{e_1} \in \left[\frac{\rho - \tau(\rho)}{1 + \rho}, \frac{\rho}{1 + \rho}\right] \qquad x_{e_2} \in \left[-\frac{\rho}{1 + \rho}, -\frac{\rho - \tau(\rho)}{1 + \rho}\right]. \tag{26}$$

In both cases

$$x_{e_1} = -x_{e_2}. \tag{27}$$

The proof of Proposition 4 is presented in the Appendix. As in the case of the no-reversal equilibria (Proposition 3), the lobbying process biases the candidates' policy choices in all reversal equilibria. However, in contrast to the no-reversal equilibria, it is possible to have reversal equilibria where both candidates have rather moderate policy preferences (close to the median policy 0) and the lobbying process leads them to implement less moderate policies that exhibit reversal, as shown in equation (26). Notice that in the case of two-candidate reversal equilibria, the set of policies that may be implemented in equilibrium is not connected. In other words, the intervals in (24) and (26) are disjoint.

Finally, we characterize the set of two-candidate hybrid equilibria.

PROPOSITION 5.   *There exist two-candidate hybrid equilibria of the electoral competition model if and only if $\rho \geq \bar{\rho}$ where $\bar{\rho}$ is defined in (13). All the two-candidate hybrid equilibria, $C(\sigma) = \{e_1, e_2\}$, where $j_1 \neq j_2$, have the following properties:*

The candidates' types either satisfy

$$j_2 = -j_1 \left(\frac{1+\rho}{1+2\rho} \right) - \rho \left(\frac{2+3\rho}{1+2\rho} \right) \tag{28}$$

and

$$j_1 \in [-1, -\rho] \qquad j_2 \in [-\tau(\rho), 0], \tag{29}$$

or

$$j_1 = -j_2 \left(\frac{1+\rho}{1+2\rho} \right) + \rho \left(\frac{2+3\rho}{1+2\rho} \right) \tag{30}$$

and

$$j_1 \in [0, \tau(\rho)] \qquad j_2 \in [\rho, 1]. \tag{31}$$

If $j_1 \in [-1, -\rho]$ and $j_2 \in [-\tau(\rho), 0]$, the equilibrium coalition choices are

$$\ell_{e_1} = \{C, R\} \qquad \ell_{e_2} = \{R\}, \tag{32}$$

and the equilibrium policy choices are

$$x_{e_1} \in \left[\max \left\{ -\frac{1-\rho}{1+2\rho}, -\frac{\rho}{1+\rho} \right\}, -\frac{\rho - \tau(\rho)}{1+\rho} \right]$$
$$x_{e_2} \in \left[\frac{\rho - \tau(\rho)}{1+\rho}, \min \left\{ \frac{1-\rho}{1+2\rho}, \frac{\rho}{1+\rho} \right\} \right]. \tag{33}$$

If instead $j_1 \in [0, \tau(\rho)]$ and $j_2 \in [\rho, 1]$ the equilibrium coalition choices are

$$\ell_{e_1} = \{L\} \qquad \ell_{e_2} = \{C, L\}. \tag{34}$$

and the equilibrium policy choices are

$$x_{e_1} \in \left[\max \left\{ -\frac{1-\rho}{1+2\rho}, -\frac{\rho}{1+\rho} \right\}, -\frac{\rho - \tau(\rho)}{1+\rho} \right]$$
$$x_{e_2} \in \left[\frac{\rho - \tau(\rho)}{1+\rho}, \min \left\{ \frac{1-\rho}{1+2\rho}, \frac{\rho}{1+\rho} \right\} \right]. \tag{35}$$

In both cases

$$x_{e_1} = -x_{e_2}. \tag{36}$$

The proof of Proposition 5 is presented in the Appendix. Notice that, in contrast to the no-reversal and reversal equilibria, hybrid equilibria do not exist for every value of $\rho > 0$. The distinctive feature of these equilibria is that although the types of both candidates are on the same side of the median policy, their equilibrium policy choices are symmetrically located around the median. Hence, unlike in the two classes of equilibria characterized in Propositions 3 and 4, the lobbying process biases the policy choice of both candidates in the same direction.

We conclude our characterization of the set of two-candidate equilibria of the electoral competition model by presenting the equilibria where the two candidates are of the same type.[21]

PROPOSITION 6. *All two-candidate equilibria of the electoral competition model,* $C(\sigma) = \{e_1, e_2\}$, *where* $j_1 = j_2$, *have the following properties:*

The candidates' type is either $j_1 = j_2 = -\rho$ *or* $j_1 = j_2 = \rho$.

If $j_1 = j_2 = -\rho$ *the equilibrium coalition choices are* $\ell_{e_1} = \ell_{e_2} = \{C, R\}$, *and the equilibrium policy choices are:* $x_{e_1} = x_{e_2} = 0$.

If instead $j_1 = j_2 = \rho$ *the equilibrium coalition choices are* $\ell_{e_1} = \ell_{e_2} = \{L, C\}$ *and the equilibrium policy choices are:* $x_{e_1} = x_{e_2} = 0$.

The proof of Proposition 6 is presented in the Appendix. Intuitively, the reason why there does not exist an equilibrium with two identical candidates that through the lobbying process implement a policy that differs from the median policy is that in this case a candidate with policy preferences equal to ρ or $-\rho$ can enter the electoral competition and win with probability 1. The reason why such a candidate would win is that although following the deviation we are considering a three-candidate equilibrium, two candidates are identical and hence only two policies may be implemented. Because, from Proposition 2, a citizen never votes for the candidate that implements his least-preferred policy, we conclude that in this situation citizens vote sincerely. Hence, the candidate that implements the median policy receives the majority of the votes.

4. Discussion

To analyze the full set of implications of our model we begin by characterizing the set of equilibria of the benchmark model where lobbying is not allowed. This

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21. The policy choices in this type of two-candidate equilibria coincide with the policy choices that would arise in all one-candidate equilibria of the model.

analysis is based on Besley and Coate (1997). Consistently with the focus of our previous analysis we restrict attention to the set of two-candidate equilibria.

When lobbying is not allowed (or equivalently when $\lambda = 0$), there exists a continuum of two-candidate equilibria. These equilibria are such that the two candidates who run for election have equal probabilities of winning and, if elected, implement policies that are symmetric around the median policy 0 (see Proposition 7 in Besley and Coate (1997) or Lemma 3 herein for the case when $\lambda = 0$). The set of policies that can be implemented in equilibrium is the entire policy space X.[22]

This characterization of the set of two-candidate equilibria survives in the citizen-candidate model with exogenous lobbying of Besley and Coate (2001). In particular, they show that it is possible to construct equilibria of such a model where the policy choices coincide with the ones that would emerge in the equilibria of the citizen-candidate model without lobbying. The equilibria of the two models are, however, different with respect to the identity of the elected candidate who implements such policies. In particular, in the model with exogenous lobbying citizens neutralize the influence of lobbies over policy by strategically electing a candidate with offsetting preferences.

We can now discuss the main implications of our analysis.

REMARK 1.   Lobbying induces policy compromise.

In all the equilibria of our model the policy outcome is a compromise between the policy most preferred by the elected candidate and the policies most preferred by the lobbies that participate in the policy-making process. This is a natural consequence of lobbying. A similar result is derived by Grossman and Helpman (1996) and Besley and Coate (2001).

REMARK 2.   Not all lobbies participate in the policy-making process.

In all the equilibria of our model no candidate ever includes all lobbies in the policy-making process. In equilibrium, there is always at least one lobby that is excluded from the bargaining process that determines the policy outcome. This is the sense in which lobbying is endogenous in our model. This implication of our analysis is consistent with the evidence presented by Wright (1996). According to Wright, many of the existing lobbies in the United States are often dormant, raising and spending no money at all. For example, between 1991 and 1992, 35% of all

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22. This characterization differs slightly from the one of Besley and Coate (1997). We assume that the cost of running for office δ is a monetary cost. Therefore, when $\lambda = 0$ this cost does not enter the payoff of a potential candidate. Besley and Coate, instead, assume that δ is a utility cost. The set of policies that can be implemented in equilibrium is then the entire policy space X with the exception of a symmetric interval around the median policy 0, whose size depends on δ. When this cost is arbitrarily small (that is $\delta \to 0$) every policy $x \in X$ can be implemented in equilibrium.

registered lobbies in the United States spent zero dollars (Wright 1996, p. 125). The fact that not all lobbies participate in the policy-making process is a key feature of our approach that distinguishes it from the menu-auction approach to lobbying (e.g., Grossman and Helpman 1996; Besley and Coate 2001), whereby assumption all lobbies participate in the policy-making process.[23]

REMARK 3.   Lobbying matters.

In our model, even though the policy preferences of all potential candidates span the entire policy space, the lobbying process reduces the set of policies that can be implemented in equilibrium. This is the sense in which lobbying matters in our model. For example, as discussed at the beginning of this section, there exists a two-candidate equilibrium in the model without lobbying where, if elected, the two candidates implement policies $x = -1$ and $x = 1$, respectively. As Corollary 1 shows, this is not an equilibrium in the model with endogenous lobbying. As discussed, this distinguishes our framework from the one of Besley and Coate (2001), where (exogenous) lobbying can have no effect on equilibrium policy outcomes. In their model, the lobbies that participate in the policy-making process can be arbitrarily chosen to guarantee that any feasible policy is implementable in equilibrium. In our model, lobbies are endogenously selected to participate in the policy-making process by the elected candidate, and not all feasible policies can be implemented in equilibrium.

REMARK 4.   Lobbying biases the outcome of the policy-making process toward the center of the policy space.

A key feature of all two-candidate equilibria without reversal is that, although the two candidates who run for office are citizens with relatively extreme policy preferences, the lobbying process induces them to implement policies that are biased toward the center of the policy space.[24] The reason for this result is that in equilibrium, elected candidates always include in their bargaining coalition lobbies whose policy preferences are on the opposite end of the policy spectrum than their own preferences. This implication of our model is consistent with the suggestive evidence presented by Austen-Smith and Wright (1994). The empirical

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23.   Although in this article we restrict attention to the case where there are only three potential lobbies, we believe that increasing the number of lobbies would only complicate the analysis without affecting our main results.

24.   We argue that equilibria that display any form of reversal are pathological. In reality, the political process has means to discipline candidates so as to prevent reversal. For example, we may think of political parties as playing such a role (e.g., Levy 2004; Morelli 2004). Alternatively, reelection concerns for elected candidates may also prevent reversal. Even though our model abstracts from the role of parties as well as repeated elections, these considerations lead us to focus on equilibria without reversal. For an alternative specification of our model where reversal never occurs in equilibrium, see Section 5.3.

findings of Austen-Smith and Wright are that, in the United States, special interest groups often lobby legislators who are predisposed to vote against their favored positions.[25]

In our model, even though the policy preferences of lobbies are relatively extreme, lobbying has a moderating effect on policy, and extreme policies never emerge as an equilibrium outcome of the political process. This result distinguishes our model from other existing models where lobbying has a tendency to induces policy outcomes that are relatively extreme (Austen-Smith 1987; Baron 1994; Groseclose and Snyder 1996; Grossman and Helpman 1996). The key difference with this literature is that in our model the entry of candidates is endogenous. Thus, lobbying affects the type of citizens who choose to run for elections as well as the policy choices of the elected policy-makers.

5. Robustness

In this section we consider a number of extensions of our model and assess the robustness of the results we derived in Section 3. In particular, we first consider equilibria with more than two candidates. We then analyze the implications of our model when citizens vote sincerely rather than strategically. We conclude by exploring an alternative specification of the utility function where citizens value only monetary contributions received by the lobby whose policy preferences are most similar to their own.

5.1. Multicandidate Equilibria

We consider the characterization of the equilibria of our model where the set of candidates $\mathcal{C}(\sigma)$ contains more than two elements. We show that the set of equilibrium policies does not depend on the number of candidates. In particular, the set of equilibrium policies with more than two candidates is the same as when there are only two candidates. Moreover, in any equilibrium, at most two different types of citizens run for office. This implies that no more than two distinct policies can be implemented in any multicandidate equilibrium with positive probability.[26] Hence, the implications of our analysis with respect to equilibrium policies are general and do not depend on whether we restrict attention to two-candidate equilibria.

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25.   One of the conclusions of the empirical analysis of Austen-Smith and Wright (1994, p. 40), who consider data on the activities of lobbying groups in the Supreme Court nomination of Robert Bork debated by the U.S. Senate in 1987, is that: "Other things being equal, groups tended to lobby 'unfriendly' senators, not those who were predisposed to vote their way." Clearly, this evidence is neither conclusive nor general and it is easy to think of other situations where lobbies target decision makers with policy preferences similar to their own. We discuss this issue further in Section 5.3.

26.   This result is analogous to the one obtained by Besley and Coate (1997) in their model of electoral competition without lobbying.

Denote n_e the cardinality of the equilibrium set of candidates $\mathcal{C}(\sigma)$ and $n_j(n_e)$ the cardinality of the corresponding set of candidates' types.

PROPOSITION 7. *All the equilibria of the electoral competition model are such that $n_j(n_e) \leq 2$ for every n_e.*

The proof of Proposition 7 is presented in the Appendix. Because candidates of the same type, if elected, implement the same policy, a direct implication of Proposition 7 is that for any equilibrium number of candidates no more than two distinct policies can be implemented with positive probability. This implies that the characterization of the sets of equilibrium policies we derived in Propositions 3, 4, 5, and 6 also applies to the general case of multi-candidate equilibria.

The following is an example of a three-candidate equilibrium. Two candidates of type $-\rho$ and one candidate of type ρ enter the electoral competition and each is elected with probability $1/3$. If elected each candidate of type $-\rho$ selects the coalition $\{C, R\}$, while the candidate of type ρ selects the coalition $\{L, C\}$. In this equilibrium each elected candidate implements the median policy 0. Similar examples can be constructed for any number of candidates n_e where the only equilibrium policy is the median of the distribution of citizens' most preferred policies.[27]

The following are examples of four-candidate equilibria where two distinct policies are implemented with positive probability. Two candidates of type $j \in [-1, -\rho)$ and two candidates of type $j' \in (\rho, 1]$, with $j = -j'$, enter the electoral competition and each is elected with probability $1/4$. If elected each candidate of type j selects the coalition $\{C, R\}$ and implements the equilibrium policy $x \in [-(1 - \rho)/(1 + 2\rho), 0)$, while each candidate of type j' selects the coalition $\{L, C\}$ and implements the equilibrium policy $x' \in (0, (1 - \rho)/(1 + 2\rho)]$ where $x = -x'$. Similar examples can be constructed for any even number of candidates where half of the candidates implements one policy and the other half implements the symmetric policy (around the median).[28]

5.2. Sincere Voting

In our analysis, following Besley and Coate (1997, 2001), we have assumed that citizens vote strategically; that is, each citizen makes his voting decision so as to maximize his expected utility given the voting decisions of all other citizens.

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27. Obviously, there exists an upper bound on n_e due to the fact that each candidate has to pay the cost of running δ and the probability of winning, and hence the expected returns from holding office, decrease with the number of candidates.

28. There are no equilibria with an odd number of candidates where two distinct policies are implemented with positive probability. In this case, it would be impossible to satisfy the equilibrium condition that all candidates win the election with equal probability.

We now consider an alternative specification where citizens vote sincerely. As in Osborne and Slivinski (1996) we take sincere voting to imply that citizens vote for the candidate who, if elected, implements the policy that is closer to their ideal policy. Moreover, if the same policy x is implemented by k candidates, each of the candidates receives the fraction $1/k$ of the votes of the citizens whose ideal policies are closer to x than to any policy implemented by any other candidate.

The main implication of sincere voting for the set of two-candidate equilibria is that, depending on the distribution F of citizens's types, the equilibrium policies of the two candidates cannot be too distant from the median policy. With sincere voting, an entrant who if elected implements the median policy always receives the votes of citizens with moderate preferences. This implies that there may not exist an equilibrium such that the two candidates, if elected, implement policies that are too distant from the median. Lobbying, however, has a moderating effect on equilibrium policies and whether or not sincere voting affects the set of equilibrium policies depends on the parameters of the model (in particular ρ and F).

Another implication of sincere voting is that there is no two-candidate equilibrium where each candidate, if elected, implements the median policy. An entrant who, if elected, implements a policy very close to the median (either to the left or the right) would in fact win the election.

The following proposition characterizes the set of two-candidate no-reversal equilibria when citizens vote sincerely.[29]

PROPOSITION 8. *All two-candidate no-reversal equilibria of the electoral competition model with sincere voting, $C(\sigma) = \{e_1, e_2\}$, have the following properties:*

(i) Suppose

$$-2\,F^{-1}(1/3) \;\leq\; \frac{1-\rho}{1+2\,\rho}. \tag{37}$$

Then the candidates' types are

$$j_1 \in \left(2\,F^{-1}(1/3)\,(1+2\,\rho) - \rho,\, -\rho\right) \quad j_2 \in \left(\rho,\, -2\,F^{-1}(1/3)\,(1+2\,\rho) + \rho\right),$$

and

$$j_1 = -j_2.$$

The equilibrium coalition choices are

$$\ell_{e_1} = \{C, R\} \qquad \ell_{e_2} = \{L, C\},$$

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29. The characterization of the set of two-candidate reversal and hybrid equilibria with sincere voting is similar and therefore omitted.

The equilibrium policy choices are

$$x_{e_1} \in (2\, F^{-1}(1/3), 0) \qquad x_{e_2} \in (0, -2\, F^{-1}(1/3)),$$

and

$$x_{e_1} = -x_{e_2}.$$

(ii) Suppose instead

$$-2\, F^{-1}(1/3) > \frac{1 - \rho}{1 + 2\,\rho}. \tag{38}$$

Then the candidates' types are

$$j_1 \in [-1, -\rho)\,, \qquad j_2 \in (\rho, 1]\,, \quad and \quad j_1 = -j_2.$$

The equilibrium coalition choices are

$$\ell_{e_1} = \{C, R\} \quad and \quad \ell_{e_2} = \{L, C\}.$$

The equilibrium policy choices are

$$x_{e_1} \in \left[-\frac{1-\rho}{1+2\,\rho}, 0\right), \qquad x_{e_2} \in \left(0, \frac{1-\rho}{1+2\,\rho}\right], \quad and \quad x_{e_1} = -x_{e_2}.$$

The proof of Proposition 8 is presented in the Appendix. Notice that for the case where condition (38) is satisfied the equilibrium characterization is identical to the one presented in Proposition 3 for the model with strategic voting, except for the fact that the equilibria where two citizens, if elected, implement the median policy 0 no longer exist. If on the other hand condition (37) is satisfied the set of equilibrium policies under sincere voting is smaller than under strategic voting.[30] In other words, sincere voting has a further moderating effect on the set of equilibrium policies. Notice, however, that all the implications of our analysis discussed in Remarks 1–4 in Section 4 still apply. Hence, the main conclusions of our analysis for the set of two-candidate equilibria are robust to replacing strategic voting with sincere voting.

An interesting difference between the model with strategic voting and the one with sincere voting is that if citizens vote sincerely there exist multicandidate equilibria where more than two distinct policies are implemented in equilibrium with positive probability. This is not true when citizens vote strategically as shown

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30.   This is, for example, the case if F is uniform on the policy space $X = [-1, 1]$ and $\rho = 1/8$. In this case, the candidates' types are $j_1 \in (-23/24, -1/8)$ and $j_2 \in (1/8, 23/24)$ and the equilibrium policy choices are $x_{e_1} \in (-2/3, 0)$ and $x_{e_2} \in (0, 2/3)$, where $j_1 = -j_2$ and $x_{e_1} = -x_{e_2}$.

in Proposition 7. To illustrate this point consider the following example of a three-candidate equilibrium where three distinct policies are implemented with positive probability. Suppose that condition (37) is satisfied. Then there exists an equilibrium where three citizens of types $j_1 = 2 F^{-1}(1/3) (1+2\rho) - \rho$, $j_2 = \rho$, and $j_3 = -2 F^{-1}(1/3) (1+2\rho) + \rho$ enter the electoral competition, win with probability 1/3, and, if elected, implement policies $x_1 = 2 F^{-1}(1/3)$, $x_2 = 0$, and $x_3 = -2 F^{-1}(1/3)$, respectively.[31]

5.3. Tainted Money

In our analysis we have assumed that all elected candidates value contributions received from lobbies in the same way, regardless of the lobbies' and the candidates' policy preferences. We now consider an alternative specification where elected candidates value only contributions received from the lobby whose policy preferences are closest to their own. In particular, we specify the utility function of citizen i of type j to be

$$U(x, y_i, j) = -(x - j)^2 + \sum_{h \in \mathcal{L}} \lambda^h y_i^h, \qquad (39)$$

where y_i^h denotes contributions received by individual i from lobby $h \in \mathcal{L} = \{L, C.R\}$ and for all $h, h' \in \mathcal{L}$, $h \neq h'$

$$\lambda^h = \begin{cases} \lambda & \text{if } |j - l_h| \leq |j - l_{h'}| \\ 0 & \text{otherwise.} \end{cases} \qquad (40)$$

Under this alternative specification of preferences all two-candidate equilibria display no reversal. Moreover, in all two-candidate equilibria, any elected candidate chooses the bargaining coalition that includes only the lobby whose most preferred policy is closest to his own. This implies that in two-candidate equilibria where $j_1 \in [-1, -1/2]$ and $j_2 \in [1/2, 1]$ the equilibrium coalition and policy choices are $\ell_{e_1} = \{L\}$ and $\ell_{e_2} = \{R\}$, and $x_{e_1} \in [-1, -(1 + 2\rho)/(2 + 2\rho)]$ and $x_{e_2} \in [(1 + 2\rho)/(2 + 2\rho), 1]$, respectively. On the other hand, in two-candidate equilibria where $j_1 \in [-1/2, 0]$ and $j_2 \in [0, 1/2]$ the equilibrium coalition and policy choices are $\ell_{e_1} = \{C\}$ and $\ell_{e_2} = \{C\}$, and $x_{e_1} \in [-1/(2 + 2\rho), 0]$ and $x_{e_2} \in [01/(2 + 2\rho)]$, respectively.[32]

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31. If F is uniform on the policy space $X = [-1, 1]$ and $\rho = 1/8$, the candidates' types are $j_1 = -23/24$, $j_2 = 1/8$, and $j_3 = 23/24$ and the equilibrium policy choices are $x_1 = -2/3$, $x_2 = 0$, and $x_3 = 2/3$.

32. The proof of this equilibrium characterization follows closely the derivations in Section 3 and is therefore omitted.

Several observations are in order. First, even under this alternative specification of preferences it is still the case that lobbying induces policy compromise and not all lobbies participate in the policy-making process (cf. Remarks 1 and 2 in Section 4). Moreover, lobbying matters (cf. Remark 3). The lobbying process reduces the set of policies that can be implemented in equilibrium because policies in the intervals $[-(1 + 2\rho)/(2 + 2\rho), -1/(2 + 2\rho)]$ and $[1/(2+2\rho), (1+2\rho)/(2+2\rho)]$ are not equilibrium policies. However, lobbying moderates policy outcomes only to the extent that relatively moderate candidates are willing to choose policies that are even more moderate by compromising with the lobby at the median of the policy space (cf. Remark 4). On the other hand, there exist equilibria where the lobbying process induces elected candidates whose types are relatively extreme to implement policies that are even more extreme.

Clearly, the specifications of preferences we have considered here represent two polar cases. At one extreme "money is money," in the sense that all contributions have the same marginal value for elected candidates regardless of the political orientation of the contributors. In this case, candidates obtain transfers from lobbies that are relatively far from them on the policy spectrum, because their willingness to pay is the highest. Hence, lobbying has a moderating effect on policy. At the other extreme "money is tainted," in the sense that elected candidates value only contributions from lobbies with political orientations that are similar to their own. In this case, candidates obtain transfers from lobbies that are relatively close to them on the policy spectrum because transfers from other lobbies have no value to them. Hence, to the extent that the policy preferences of lobbies are extreme, lobbying yields relatively extreme policies. There is a continuum of specifications of preferences that lie between these two extremes. For such intermediate cases both mechanisms would be at work and the ultimate effect of lobbying on policy would depend on the details of the model.[33]

6. Concluding Remarks

In this article we have focused exclusively on the role of lobbying activities that target elected policy-makers. In reality lobbies also engage in a number of other activities that range from providing campaign contributions to influencing, mobilizing, and informing voters (Grossman and Helpman 2001). Although the relative importance of each of these activities vary across political institutions and through time, we believe that trying to influence the behavior of elected politicians is a primary goal of many lobbies in all democratic systems.

Part of this lobbying activity is covert in nature and may therefore have distortionary effects on policy; however, there also exist legal ways to make transfers

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33. The coexistence of these two mechanisms might help interpret the mixed evidence on lobbying activities (Wright 1996).

to elected politicians—for example, providing job opportunities at the end of their political career (Diermeier, Keane, and Merlo 2005). These considerations raise a number of challenges both at the theoretical and empirical level that are outside of the scope of this article. They represent, however, the next step to further our understanding of lobbying in modern democracies.

Appendix

Proof of Lemma 1. From equations (3) and (4) the objective function in (8) is strictly concave. The first-order conditions of problem (8) are

$$(x - E) + \rho \sum_{h \in \ell} (x - l_h) = 0. \tag{A.1}$$

Then the unique solution of equation (A.1) is (10).   □

Proof of Lemma 2. Given that our model is completely symmetric around the median policy 0, we prove the result for the case in which the most preferred policy choice E by the elected candidate P^E is such that $E \geq 0$. The case $E \leq 0$ is completely symmetric, and therefore the proof is omitted.

Notice first that P^E's optimal policy choices for every $\ell \in \Delta$—the solution to problem (8) above—are

$$x_{PE}(\emptyset) = E, \quad x_{PE}(\{L\}) = \frac{E - \rho}{1 + \rho},$$

$$x_{PE}(\{C\}) = \frac{E}{1 + \rho}, \tag{A.2}$$

$$x_{PE}(\{R\}) = \frac{E + \rho}{1 + \rho},$$

together with

$$x_{PE}(\{L, C\}) = \frac{E - \rho}{1 + 2\rho}, \quad x_{PE}(\{L, R\}) = \frac{E}{1 + 2\rho},$$

$$x_{PE}(\{C, R\}) = \frac{E + \rho}{1 + 2\rho} \tag{A.3}$$

and

$$x_{PE}(\{L, C, R\}) = \frac{E}{1 + 3\rho}. \tag{A.4}$$

It is now possible to evaluate the elected candidate's payoff for every coalition choice $\ell \in \Delta$ using (A.2), (A.3), and (A.4). These payoffs are

$$u(x_{PE}(\emptyset), E) = 0, \tag{A.5}$$

$$u(x_{PE}(\{L\}), E) + \lambda W_{\{L\}}(x_{PE}(\{L\}), E) = \left(\frac{\rho}{1+\rho}\right)(1+E)^2, \quad (A.6)$$

$$u(x_{PE}(\{C\}), E) + \lambda W_{\{C\}}(x_{PE}(\{C\}), E) = \left(\frac{\rho}{1+\rho}\right)(E)^2, \quad (A.7)$$

and

$$u(x_{PE}(\{R\}), E) + \lambda W_{\{R\}}(x_{PE}(\{R\}), E) = \left(\frac{\rho}{1+\rho}\right)(E-1)^2. \quad (A.8)$$

The payoff in (A.6) weakly dominates all the payoffs in (A.5), (A.7), and (A.8) for every $E \in [0, 1]$ and is therefore the only relevant payoff among the one computed above. The elected candidate's payoffs for the remaining coalitions $\ell \in \Delta$ are

$$u(x_{PE}(\{L, C\}), E) + \lambda W_{\{L,C\}}(x_{PE}(\{L, C\}), E)$$
$$= \left(\frac{\rho}{1+2\rho}\right)[1 + 4E + 4(E)^2], \quad (A.9)$$

$$u(x_{PE}(\{L, R\}), E) + \lambda W_{\{L,R\}}(x_{PE}(\{L, R\}), E)$$
$$= \left(\frac{\rho}{1+2\rho}\right)4(E)^2, \quad (A.10)$$

$$u(x_{PE}(\{C, R\}), E) + \lambda W_{\{C,R\}}(x_{PE}(\{C, R\}), E)$$
$$= \left(\frac{\rho}{1+2\rho}\right)[1 - 4E + 4(E)^2], \quad (A.11)$$

and finally

$$u(x_{PE}(\{L, C, R\}), E) + \lambda W_{\{L,C,R\}}(x_{PE}(\{L, C, R\}), E)$$
$$= \left(\frac{\rho}{1+3\rho}\right)9(E)^2. \quad (A.12)$$

The payoff in (A.9) weakly dominates all the payoffs in (A.10), (A.11), and (A.12) for every $E \in [0, 1]$. Therefore the only relevant comparison is the one between the payoffs in (A.6) and in (A.9).

The payoff in (A.9) is greater or equal than the payoff in (A.6) for every $E \in [0, \tau(\rho)]$ where $\tau(\rho)$ is defined in (12). In other words, for every $E \in [0, \tau(\rho)]$ the coalition choice that, in this case, solves problem (9) is $\ell_{PE} = \{L, C\}$. Conversely, the payoff in (A.6) is greater or equal than the payoff in (A.9)

for every $E \in [\tau(\rho), 1]$. In other words, the coalition choice that, in this case, solves problem (9) is $\ell_{PE} = \{L\}$.   □

Proof of Proposition 2.   Assume by way of contradiction that there exists an equilibrium of the voting subgame that survives one round of elimination of weakly dominated strategies and is such that citizen i^j votes for candidate $\underline{e}$. The voting profiles of all the citizens but i^j can be partitioned into two sets. The set of profiles such that citizen i^j is not pivotal and the set of profiles such that citizen i^j is pivotal. Citizen i^j is pivotal if, when $\sigma(i^j) = \underline{e}$, candidate $\underline{e}$ is elected, whereas when $\gamma(i^j) \neq \underline{e}$ the elected candidate is $e \in \mathcal{C}(\sigma)$ with $e \neq \underline{e}$.

   If citizen i^j is not pivotal then citizen i^j's payoff is the same whatever his vote. If instead citizen i^j is pivotal then by definition (16) of $\underline{e}$ citizen i^j's payoff is weakly lower if his vote is $\gamma(i^j) = \underline{e}$ than if it is $\gamma(i^j) \neq \underline{e}$. This implies that $\gamma(i^j) = \underline{e}$ is a weakly dominated strategy. This is clearly a contradiction to the hypothesis that the equilibrium of the voting subgames survives one round of elimination of weakly dominated strategy.   □

Proof of Lemma 3.   Assume by way of contradiction that the two-candidate equilibria of the electoral competition model, $\mathcal{C}(\sigma) = \{e_1, e_2\}$, where $j_1 \neq j_2$, are such that

$$x_{e_1}(\ell_{e_1}) \neq -x_{e_2}(\ell_{e_2}).$$

In particular, without any loss of generality we assume that

$$\left| x_{e_1}(\ell_{e_1}) \right| < \left| x_{e_2}(\ell_{e_2}) \right|. \tag{A.13}$$

By Proposition 2 all citizens will vote sincerely. In other words, the citizens of type j^*, where

$$j^* = \frac{x_{e_1}(\ell_{e_1}) + x_{e_2}(\ell_{e_2})}{2}$$

are indifferent between voting for one candidate or the other. All the citizens of type $j > j^*$ in equilibrium will vote for the candidate whose policy $x_e(\ell_e) > j^*$ and all the citizens of type $j < j^*$ in equilibrium will vote for the other candidate. This implication, together with assumption (A.13), implies that more than half of the population will vote for candidate e_1. Therefore, candidate e_1 wins the vote with probability 1. This implies that candidate e_2 has a profitable deviation. By not running for office he does not change the policy choice but increases his payoff of the cost of running $\delta > 0$. This contradicts the hypothesis that there exists a two candidate equilibrium where (A.13) is satisfied.   □

Proof of Proposition 3.   We start from (18). Proposition 1 implies that the policy selected by the elected candidate P^E is equal to the median policy choice 0 if and only if $E = -\rho$ and $E = \rho$. Because, from Proposition 1, the optimal policy

choice is monotonic increasing in E if $E \leq 0$ and monotonic decreasing in E if $E \geq 0$, we do not observe any policy reversal for $j_1 \in [-1, -\rho]$ and $j_2 \in [\rho, 1]$. From the definition (12) of $\tau(\rho)$ for every $\rho \in (0, 1]$ we have $0 < \tau(\rho) < \rho$ and $\lim_{\rho \to 0} \tau(\rho) = 0$. This implies that, from Proposition 1, for $j_1 \in [-1, -\rho]$ and $j_2 \in [\rho, 1]$ the equilibrium policy choices are such that

$$x_{e_1} = \frac{j_1 + \rho}{1 + 2\rho} < 0, \qquad x_{e_2} = \frac{j_2 - \rho}{1 + 2\rho} > 0. \tag{A.14}$$

Lemma 3 and the symmetry of the policy choices (A.14) around the median policy 0 yield $j_1 = -j_2$, which completes the proof of (18).

Condition (19) follows directly from Proposition 1 and the observation that for every $\rho \in (0, 1]$, $\tau(\rho) < \rho$. Lemma 3 and the policy choices in (A.14) imply (20) and (21).

If δ is small enough, the two candidates of types j_1 and j_2 that satisfy (18) run for office and are elected with probability $1/2$. This implies that neither candidate is willing to withdraw from the electoral race, because both

$$u(x_{e_1}, j_1) + \lambda[W_{\{C,R\}}(x_{e_1}, j_1) - \delta] - u(x_{e_2}, j_1)$$

and

$$u(x_{e_1}, j_1) + \lambda[W_{\{C,R\}}(x_{e_1}, j_1) - \delta] - u(x_{e_2}, j_1)$$

are strictly positive if δ is small enough.

To conclude the proof we still need to specify the out-of-equilibrium path behavior of nonpivotal voters such that no other candidate is willing to enter the electoral competition. We assume that if a candidate e' enters the electoral competition no citizen will vote for candidate e' and all citizens will vote for the one of the two candidates e_1 and e_2 that chooses the policy choice that is closer to each citizen's type among $\{x_{e_1}, x_{e_2}\}$. In this case citizen e' cannot win the vote and cannot affect the outcome of the election and the equilibrium policy choice. Therefore, e' is strictly better off by not running and saving the cost δ. No citizen is pivotal in determining whether candidate e' wins the election. Therefore, the specified voting behavior is compatible with strategic voting. Finally, no citizen votes for the one among the three candidates e', e_1, and e_2 that implements the least-preferred policy choice. Therefore, the specified voting behavior is compatible with Proposition 2 above. $\square$

Proof of Proposition 4. We start from condition (22). As argued in the proof of Proposition 3, Proposition 1 implies that the policy selected by the elected candidate P^E is equal to the median policy choice 0 if and only if $E = -\rho$ and $E = \rho$. Proposition 1 also implies that the optimal policy choice is monotonic increasing in E if $E \leq 0$, and monotonic decreasing in E if $E \geq 0$. Therefore

we observe reversal of policy choices for $j_1 \in [-\rho, 0]$ and $j_2 \in [0, \rho]$. Because $\tau(\rho) < \rho$, from Proposition 1 we need to distinguish between the case $j_1 \in [-\rho, -\tau(\rho)]$ and $j_2 \in [\tau(\rho), \rho]$ and the case $j_1 \in [-\tau(\rho), 0]$ and $j_2 \in [0, \tau(\rho)]$. In the first case, $j_1 \in [-\rho, -\tau(\rho)]$ and $j_2 \in [\tau(\rho), \rho]$, the equilibrium policy choices are such that

$$x_{e_2} = \frac{j_2 - \rho}{1 + 2\rho} < 0, \qquad x_{e_1} = \frac{j_1 + \rho}{1 + 2\rho} > 0. \tag{A.15}$$

Whereas in the second case, $j_1 \in [-\tau(\rho), 0]$ and $j_2 \in [0, \tau(\rho)]$, the equilibrium policy choices are such that

$$x_{e_2} = \frac{j_2 - \rho}{1 + \rho} < 0, \qquad x_{e_1} = \frac{j_1 + \rho}{1 + \rho} > 0. \tag{A.16}$$

Lemma 3 and the symmetry of the policy choices in (A.15) and (A.16) imply that $j_1 = -j_2$, which completes the proof of (18).

Conditions (23) and (25) follow directly from Proposition 1 and $\tau(\rho) < \rho$. Lemma 3 and the policy choices in (A.15) and (A.16) imply (24), (26), and (27).

If δ is small enough, the two candidates of types j_1 and j_2 that satisfy (18) run for office and are elected with probability $1/2$. This implies that neither candidate is willing to withdraw from the electoral race because, for δ small enough, when $j_1 \in [-\rho, -\tau(\rho)]$ and $j_2 \in [\tau(\rho), \rho]$ we have

$$\begin{aligned}
u(x_{e_1}, j_1) + \lambda[W_{\{C,R\}}(x_{e_1}, j_1) - \delta] - u(x_{e_2}, j_1) &> 0 \\
u(x_{e_2}, j_2) + \lambda[W_{\{L,C\}}(x_{e_2}, j_2) - \delta] - u(x_{e_1}, j_2) &> 0,
\end{aligned} \tag{A.17}$$

whereas when $j_1 \in [-\tau(\rho), 0]$ and $j_2 \in [0, \tau(\rho)]$ we have

$$\begin{aligned}
u(x_{e_1}, j_1) + \lambda[W_{\{R\}}(x_{e_1}, j_1) - \delta] - u(x_{e_2}, j_1) &> 0 \\
u(x_{e_2}, j_2) + \lambda[W_{\{L\}}(x_{e_2}, j_2) - \delta] - u(x_{e_1}, j_2) &> 0.
\end{aligned} \tag{A.18}$$

To conclude the proof we still need to specify the out-of-equilibrium path behavior of non-pivotal voters such that no other candidate is willing to enter the electoral competition. As in the proof of Proposition 3, we assume that if a candidate e' enters the electoral competition no citizen will vote for candidate e' and all citizens will vote for the one of the two candidates e_1 and e_2 that chooses the policy choice that is closer to each citizen's type among $\{x_{e_1}, x_{e_2}\}$. Then citizen e' cannot win the vote and cannot affect the outcome of the election and the equilibrium policy choice. Therefore, e' is strictly better off by not running and saving the cost δ. No citizen is pivotal in determining whether candidate e' wins the election. Therefore, the specified voting behavior is compatible with strategic voting. Finally, no citizen votes for the one, among the three candidates e', e_1

and e_2, that implements the least preferred policy choice. Therefore, the specified voting behavior is compatible with Proposition 2.   □

Proof of Proposition 5. Given that our model is completely symmetric around the median policy 0, we prove the result in the case $j_1 \leq j_2 \leq 0$. The case $j_2 \geq j_1 \geq 0$ is completely symmetric and therefore the proof is omitted.

Recall that by definition hybrid equilibria are such that candidate e_1 chooses a policy that does not exhibit policy reversal, and is to the left of the median policy 0, whereas candidate e_2 chooses a policy that does exhibits policy reversal, and is to the right of the median policy 0. Proposition 1 implies that for this to be the case we need $j_1 \in [-1, -\rho]$ and $j_2 \in [-\rho, 0]$. Notice first that if $j_2 \in [-\rho, -\tau(\rho)]$ Proposition 1 implies that Lemma 3 cannot hold because

$$x_{e_1} = \frac{j_1 + \rho}{1 + 2\rho} > -x_{e_2} = -\frac{j_2 + \rho}{1 + \rho}$$

for $j_1 \in [-1, -\rho]$ and $j_2 \in [-\rho, -\tau(\rho)]$. Therefore, a necessary condition for a hybrid equilibrium to exist is $j_2 \in [-\tau(\rho), 0]$.

Again, for Lemma 3 to hold we need that the policy choices x_{e_1} and x_{e_2} satisfy condition (17) or equivalently (36). For this to be the case we need that the distance from the median policy 0 of the smallest policy choice that candidate e_1 can implement is greater than or equal to the distance from the median policy 0 of the smallest policy choice that candidate e_2 can implement. From Proposition 1 this condition implies that a necessary condition for an equilibrium to exist is that ρ is such that

$$-\frac{1 - \rho}{1 + 2\rho} \leq -\frac{\rho - \tau(\rho)}{1 + \rho}. \tag{A.19}$$

In other words, from the definition (12) of $\bar{\rho}$, a necessary condition for a hybrid two-candidate equilibrium to exist is $\rho \leq \bar{\rho}$. To show that this is also a sufficient condition it is enough to observe that if $\rho = \bar{\rho}$ then $j_1 = -1$ and $j_2 = -\tau(\rho)$ is a hybrid two-candidate equilibrium of the electoral competition model that satisfies conditions (28), (29), (32), (33), and (36).

Lemma 3 also implies that the two policy choices x_{e_1} and x_{e_2} must satisfy

$$x_{e_1} = \frac{j_1 + \rho}{1 + 2\rho} = -x_{e_2} = -\frac{j_2 + \rho}{1 + \rho}. \tag{A.20}$$

Solving (A.20) for j_1 we obtain (28).

Condition (32) follows directly from Proposition 1. Lemma 3 and the necessary and sufficient condition for the existence of a hybrid equilibrium $\rho \leq \bar{\rho}$ imply (33) and (36). Notice that the sets in (33) are nonempty if and only if $\rho \leq \bar{\rho}$.

If δ is small enough the two candidates of types j_1 and j_2 that satisfy (28) and (29) run for office and are elected with probability $1/2$. Neither candidate is

willing to withdraw from the electoral race because, for δ small enough, when $j_1 \in [-1, -\rho]$ and $j_2 \in [-\tau(\rho), 0]$ we have

$$
\begin{aligned}
u(x_{e_1}, j_1) + \lambda[W_{\{C,R\}}(x_{e_1}, j_1) - \delta] - u(x_{e_2}, j_1) &> 0 \\
u(x_{e_2}, j_2) + \lambda[W_{\{R\}}(x_{e_2}, j_2) - \delta] - u(x_{e_1}, j_2) &> 0.
\end{aligned}
\tag{A.21}
$$

To conclude the proof we still need to specify the out-of-equilibrium path behavior of nonpivotal voters such that no other candidate is willing to enter the electoral competition. As in the proof of Propositions 3 and 4 we assume that if a candidate e' enters the electoral competition no citizen will vote for candidate e' and all citizens will vote for the one of the two candidates e_1 and e_2 that chooses the policy that is closer to each citizen's type among $\{x_{e_1}, x_{e_2}\}$. Then citizen e' cannot win the vote and cannot affect the outcome of the election and the equilibrium policy choice. Therefore, e' is strictly better off by not running and saving the cost δ. No citizen is pivotal in determining whether candidate e' wins the election. Therefore, the specified voting behavior is compatible with strategic voting. Finally, no citizen votes for the one, among the three candidates e', e_1, and e_2, that implements the least preferred policy choice. Therefore, the specified voting behavior is compatible with Proposition 2 above. $\square$

Proof of Proposition 6. Given that our model is completely symmetric around the median policy 0, we prove the result in the case $j_1 = j_2 = -\rho$. The case $j_2 = j_1 = \rho$ is completely symmetric and therefore the proof is omitted.

Proposition 1 implies that the policy selected by the elected candidates e_1 and e_2 is equal to the median policy choice if $j_1 = j_2 = -\rho$, the coalition selected by both candidates is $\ell_{e_1} = \ell_{e_2} = \{C, R\}$, and the policy choice is, of course, $x_{e_1} = x_{e_2} = 0$.

If δ is small enough the two candidates of type $j_1 = j_2 = -\rho$ run for office and are elected with probability $1/2$. Neither candidate is willing to withdraw from the electoral race because, for δ small enough, when $j_1 = j_2 = -\rho$ we have

$$
\begin{aligned}
u(0, -\rho) + \lambda[W_{\{C,R\}}(0, -\rho) - \delta] - u(0, -\rho) \\
= \lambda[W_{\{C,R\}}(0, -\rho) - \delta] > 0.
\end{aligned}
\tag{A.22}
$$

To conclude the proof we need to specify the out-of-equilibrium path behavior of nonpivotal voters such that no other candidate is willing to enter the electoral competition. If a candidate e' of type $j' \neq -\rho$ enters the electoral competition Proposition 2 implies that citizens will vote for the one among the three candidates e', e_1, and e_2, that will choose his most preferred policy choice. This is because the two candidates e_1 and e_2 choose the same policy choice and hence only two policy outcomes will be observed following the entry of e'. We assume, however, that among the candidates e_1 and e_2 all the citizens who prefer the median policy 0 to policy j' will concentrate their vote on just one candidate, for example e_1. Then

citizen e' cannot win the vote and cannot affect the outcome of the election and the equilibrium policy choice. Therefore, e' is strictly better off by not running and saving the cost δ. No citizen is pivotal in determining whether candidate e_1 or e_2 wins the election. Therefore, the specified voting behavior is compatible with strategic voting.

If instead a candidate e'' of type $j'' = -\rho$ enters the electoral competition we assume that no citizen will vote for e'' and all the citizens will vote for either e_1 or e_2. Then citizen e'' cannot win the vote and cannot affect the outcome of the election and the equilibrium policy choice. Therefore e'' is strictly better off by not running and saving the cost δ. No citizen is pivotal in determining whether candidate e'' wins the election. Therefore, the specified voting behavior is compatible with strategic voting. Finally, because all three candidates e'', e_1, and e_2 implement the same policy choice the specified voting behavior is trivially compatible with Proposition 2 above. $\square$

Proof of Proposition 7. Assume by way of contradiction that $n_j(n_e) > 2$. There exist two cases depending on whether $n_e = n_j(n_e)$ or $n_e > n_j(n_e)$. Consider the former case first. For an equilibrium with $n_j(n_e) > 2$ to exist it must be the case that each candidate in $\mathcal{C}(\sigma)$ has probability $1/n_e$ of winning the election. This clearly means that each citizen in the population of voters is *pivotal,* in the sense that each citizen by deviating and changing how he casts his vote can make one of the n_e candidate win with probability 1 rather than $1/n_e$.

Denote $\mathcal{J}$ the set of candidates types. Consider two candidates e_1 and e_2 of types j_1 and j_2, respectively, that implement contiguous but different policies x_{e_1} and x_{e_2}. Without loss in generality let these policies be $x_{e_1} < x_{e_2}$ such that there does not exist a candidate $e_3 \in \mathcal{C}(\sigma)$ that implements policy x_{e_3} such that $x_{e_1} \leq x_{e_3} \leq x_{e_2}$. Recall that by assumption every type $j \in X$ is represented in the population of citizens. Then there exists a citizen of type p such that

$$|x_{e_1} - p| = |x_{e_2} - p|. \tag{A.23}$$

Let the equilibrium be such that this citizen votes for candidate e_1. This citizen has a profitable deviation. Indeed, it is profitable for this citizen to cast his vote for candidate e_2. The equilibrium payoff of the citizen of type p is

$$-\sum_{i \in \mathcal{J}} \frac{1}{n_j} \left(x_{e_i} - p\right)^2. \tag{A.24}$$

If instead the citizen of type p deviates and casts his vote for citizen e_2, then his payoff is

$$-(x_{e_2} - p)^2. \tag{A.25}$$

By definition (A.23), whenever $n_j > 2$ there exists a type-j candidate $e_j \in \mathcal{C}(\sigma)$ such that $|x_{e_j} - p| > |x_{e_2} - p|$, therefore the payoff in (A.24) is strictly lower

than the payoff in (A.25). This clearly contradict the hypothesis that there exists an equilibrium with $n_j(n_e) > 2$.

Notice that the latter contradiction does not arise in the case in which $n_j(n_e) = 2$.

The proof of Proposition 7 in the case where $n_e > n_j(n_e)$ is similar and therefore omitted. $\square$

LEMMA A.1.   *All two-candidate equilibria of the electoral competition model with sincere voting, $\mathcal{C}(\sigma) = \{e_1, e_2\}$, are such that*

$$x_{e_1}(\ell_{e_1}) = -x_{e_2}(\ell_{e_2}), \qquad (A.26)$$

$$x_{e_1}(\ell_{e_1}) \neq 0 \quad \textit{or equivalently} \quad x_{e_2}(\ell_{e_2}) \neq 0, \qquad (A.27)$$

and

$$x_{e_1}(\ell_{e_1}) \geq 2F^{-1}(1/3) \quad \textit{or equivalently} \quad x_{e_2}(\ell_{e_2}) \leq -2F^{-1}(1/3). \quad (A.28)$$

Proof. Consider (A.26) first. Assume that $j_1 \neq j_2$ and a two-candidate equilibrium exists. Proposition 2 implies that whenever the set $\mathcal{C}(\sigma)$ contains only two elements strategic voting coincides with sincere voting. This implies that the proof of (A.26) coincides with the proof of Lemma 3 above.

Consider now (A.27). Assume by way of contradiction that there exists a two-candidate equilibrium of the electoral competition game with sincere voting such that $x_{e_1}(\ell_{e_1}) = x_{e_2}(\ell_{e_2}) = 0$. Consider now a citizen e_j of type $j = \rho + \varepsilon$ where ε is an arbitrarily small positive number. Assume this citizen decides to deviate and enter the electoral competition. The sincere voting rule implies that all citizens whose type is in the interval $[\varepsilon/(2 + 2\rho), 1]$ will vote for e_j, whereas all citizens with types in the interval $[-1, \varepsilon/(2 + 2\rho)]$ will split their votes equally between e_1 and e_2. Because approximately half of the population of citizens will vote for e_j and approximately a quarter of the population of citizens will vote for each of the candidates e_1 and e_2, candidate e_j will win the election with probability 1. Because in our environment an elected candidate earns a strictly positive payoff this deviation is profitable for citizen e_j. This contradicts the hypothesis that there exists a two-candidate equilibrium with $x_{e_1}(\ell_{e_1}) = x_{e_2}(\ell_{e_2}) = 0$.

Consider now (A.28). Assume by way of contradiction that there exists a two-candidate equilibrium of the electoral competition game with sincere voting such that $x_{e_1}(\ell_{e_1}) < 2F^{-1}(1/3)$ or equivalently from (A.26), $x_{e_2}(\ell_{e_2}) > -2F^{-1}(1/3)$. Consider now a citizen e_j of type $j = \rho$ and assume that this citizen decides to enter the electoral competition. The sincere voting rule implies that all citizens whose type is in the interval $[x_{e_1}(\ell_{e_1})/2, x_{e_2}(\ell_{e_2})/2]$ will vote for citizen e_j whereas all the citizens whose type is in the interval $[-1, x_{e_1}(\ell_{e_1})/2]$ will vote for candidate e_1 and all the citizens whose type

is in the interval $[x_{e_2}(\ell_{e_2})/2, 1]$ vote for candidate e_2. Because by assumption $x_{e_1}(\ell_{e_1}) = -x_{e_2}(\ell_{e_2}) < 2\,F^{-1}(1/3)$, we then conclude that the mass of citizens that vote for e_j is such that

$$F(x_{e_2}(\ell_{e_2})) - F(x_{e_1}(\ell_{e_1}) > \frac{1}{3}, \tag{A.29}$$

and the mass of citizens that vote for e_1 and e_2 is the same and it is equal to

$$F(x_{e_2}(\ell_{e_2})) = 1 - F(x_{e_1}(\ell_{e_1}) < \frac{1}{3}. \tag{A.30}$$

Clearly (A.29) and (A.30) imply that citizen e_j wins the election with probability 1. Because in our environment an elected candidate earns a strictly positive payoff this deviation is profitable for citizen e_j. This contradicts the hypothesis that there exists a two-candidate equilibrium with $x_{e_1}(\ell_{e_1}) = -x_{e_2}(\ell_{e_2}) < 2\,F(1/3)$. $\square$

Proof of Proposition 8. The proof follows directly from Lemma A.1, the observation that the characterization of the equilibria of the lobbying subgame in Subsection 3.1 is the same whether the citizens vote strategically or sincerely and the fact that Proposition 2 implies that when the set $\mathcal{C}(\sigma)$ includes only two candidates voting strategically is equivalent to voting sincerely. $\square$

References

Alesina, Alberto (1988). "Credibility and Policy Convergence in a Two-Party System with Rational Voters." *American Economic Review*, 78, 796–806.

Austen-Smith, David (1987). "Interest Groups, Campaign Contributions, and Probabilistic Voting." *Public Choice*, 54, 123–139.

Austen-Smith, David, and John R. Wright (1992). "Competitive Lobbying for a Legislator's Vote." *Social Choice and Welfare*, 9, 229–257.

Austen-Smith, David, and John R. Wright (1994). "Counteractive Lobbying." *American Journal of Political Science*, 38, 25–44.

Baron, David P. (1994). "Electoral Competition with Informed and Uninformed Voters." *American Political Science Review*, 88, 33–47.

Bernheim, B. Douglas, and Michael D. Whinston (1986). "Menu Auctions, Resource Allocation, and Economic Influence." *Quarterly Journal of Economics*, 101, 1–31.

Besley, Timothy, and Stephen Coate (1997). "An Economic Model of Representative Democracy." *Quarterly Journal of Economics*, 108, 85–114.

Besley, Timothy, and Stephen Coate (2001). "Lobbying and Welfare in a Representative Democracy." *Review of Economic Studies*, 68, 67–82.

Diermeier, Daniel, Michael Keane, and Antonio Merlo (2005). "A Political Economy Model of Congressional Careers." *American Economic Review*, 95, 347–373.

Diermeier, Daniel, and Antonio Merlo (2000). "Government Turnover in Parliamentary Democracies." *Journal of Economic Theory*, 94, 46–79.

Dixit, Avinash, Gene Grossman, and Elhanan Helpman (1997). "Common Agency and Coordination: General Theory and Application to Government Policy Making." *Review of Economic Studies*, 105, 752–769.

Downs, Antony (1957). *An Economic Theory of Democracy*. Harper Collins.

Groseclose, Tim, and James M. Snyder (1996). "Buying Supermajorities." *American Political Science Review*, 90, 303–315.

Grossman, Gene, and Elhanan Helpman (1994). "Protection for Sale." *American Economic Review*, 84, 833–850.

Grossman, Gene, and Elhanan Helpman (1996). "Electoral Competition and Special Interest Politics." *Review of Economic Studies*, 63, 265–286.

Grossman, Gene, and Elhanan Helpman (2001). *Special Interest Politics*. M.I.T. Press.

Hibbs, Douglas (1977). "Political Parties and Macroeconomic Policy." *American Political Science Review*, 71, 1467–1487.

Levy, Gilat (2004). "A Model of Political Parties." *Journal of Economic Theory*, 115, 250–277.

Morelli, Massimo (2004). "Party Formation and Policy Outcomes under Different Electoral Systems." *Review of Economic Studies*, 71, 829–853.

Osborne, Martin, and Al Slivinski (1996). "A Model of Political Competition with Citizen Candidates." *Quarterly Journal of Economics*, 111, 65–96.

Persson, Torsten, and Elhanan Helpman (1998). "Lobbying and Legislative Bargaining." NBER working paper no. 6589.

Persson, Torsten, and Guido Tabellini (2000). *Political Economics: Explaining Economic Policy*. M.I.T. Press.

Tullock, Gordon (1967). "The Welfare Costs of Tariffs, Monopolies and Theft." *Western Economic Journal*, 5, 224–232.

Wittman, Donald (1977). "Candidates with Policy Preferences: A Dynamic Model." *Journal of Economic Theory*, 14, 180–189.

Wright, John R. (1996). *Interest Groups and Congress*. Allyn & Bacon.