Learning, Wage Dynamics, and Firm-Specific Human Capital

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We introduce a dynamic and fully strategic model of wage determination in the presence of firm-specific human capital. In this model, human capital is interpreted as information. We show that equilibrium exists and is efficient and that it gives rise to a unique distribution of the social surplus. We show further that the equilibrium wage is determined by three factors. Consideration of these factors allows us to determine when and how the market mechanism enables the worker to capture some of the returns to firm-specific human capital. We relate our findings to the ongoing empirical debate concerning the return to tenure.

I. Introduction

Whether wages increase with tenure within a given employment relationship is an open question in the empirical analysis of labor markets.

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Some recent studies, such as Abraham and Farber (1987) and Altonji and Shakotko (1987), find that while wages do increase with total job market experience, they are not systematically related to tenure on the current job. In other words, there is little or no return to tenure beyond that attributable to general experience. Others, such as Topel (1991), argue that when econometric problems are correctly addressed, evidence of a strong positive relationship between wages and tenure emerges. In other words, there is a significant return to tenure over and above that attributable to general experience.

It is tempting to interpret these findings in terms of the accumulation of general and specific human capital. Indeed, general human capital can be expected to accumulate with total job market experience. Wages will then increase with experience to the extent that the worker is able to capture some of the return to this capital. Similarly, specific human capital can be expected to accumulate with tenure with a given employer. Wages will then increase with tenure to the extent that a worker is able to capture some of the return to this capital.

There is, however, a difficulty with such an interpretation. General human capital enhances a worker's productivity with every potential employer. Competition can therefore be expected to ensure that she receives at least some of the return to such capital. By contrast, specific human capital enhances the worker's productivity only in a single firm. There is therefore no competition for such capital, and it is by no means clear what force will ensure that she receives any return on it.

The purpose of the present paper is to develop a dynamic model of wage determination in the presence of firm-specific human capital. In this model, there are two firms and one worker. Each period divides into four stages: (i) both firms propose a wage, (ii) the worker chooses between the two firms, (iii) production takes place in the chosen firm, and (iv) the output of the chosen firm is observed and the wage is paid. The wage negotiation and production cycle then recommences. Human capital accumulates at a rate that depends only on the current stock of human capital and on the choice of employer. In particular, market participants can influence the rate of accumulation of human capital only insofar as they can influence the worker's choice of employer.

Following Jovanovic (1979), we adopt an informational interpretation of human capital. More explicitly, we assume that both the productivity of the worker in firm 1 and her productivity in firm 2 are initially unknown, and we interpret information about her productivity as human capital. We assume furthermore that her productivity in firm 1 is independent of her productivity in firm 2 and remains so over time. In other words, we assume that all human capital is firm specific. This allows us to focus clearly on whether and, if so, how the market mechanism allows a worker to capture any of the return to her specific human capital. Finally, we assume that production is stochastic. This ensures that human capital accumulates slowly over time.

More precisely, we shall consider two models. Three results are common to these two models. First, equilibrium is efficient in the sense that it maximizes the payoff of the grand coalition consisting of all three market participants. This result is striking in that it is obtained even though there are no long-term employment contracts. It is also robust to many variations in the model.[1] Second, equilibrium exists and is unique. Third, the current employer pays the worker the full value of the match with the alternative employer. More precisely, there are three components to the wage: (1) the worker's expected product in the alternative match, (2) a premium reflecting the accumulation of human capital specific to the alternative match that the worker forgoes by working for the current employer, and (3) a reduction reflecting the accumulation of human capital specific to the alternative match that the worker obtains by working for the current employer. The second component captures the idea that the worker might like to quit her current employer, work for the alternative employer for a spell (thereby driving up her outside option), and then return to her current employer. The third component captures the idea that the current employer can capture the dynamic gain to the worker from her current employment.

The remaining results depend on which of the two models is considered. In the first, or basic, model, there is only one task that the worker can undertake in each firm, and the current match does not result in any learning about the quality of the alternative match. In this model, the worker is paid her static outside option, namely her expected product in the alternative match, and the wage is therefore constant through tenure. The wage does, however, change on match termination. Indeed, it changes discontinuously, and it can go either up or down.

These results can be explained as follows. The worker is paid her

[1] The result appears to depend only on the symmetry of information, the fact that there are only two firms, and the fact that all market participants have the same discount rate. The result will still hold if the worker's characteristic in each firm can take on many values, as opposed to the two values allowed in the present paper; the characteristic of the worker in one firm is correlated with her characteristic in the other firm; and observation of the production, or production vector, of the worker in one firm reveals information about her characteristic in the other firm. In particular, it will still hold in a model in which there is general as well as specific human capital.

expected product in the alternative match because, while she does forgo the accumulation of human capital specific to the alternative match in the course of the current match, such capital has no value within the context of the alternative match. Hence the second component of the wage is zero. Moreover, no learning about the quality of the alternative match takes place in the course of the current match. Hence the third component of the wage is zero too. The wage is constant through tenure because the worker is paid her static outside option, this option depends only on her product in the alternative match, and the market's estimate of this product does not change in the course of the current match. Finally, discontinuities in the wage occur on match termination because, although the wage is the expected product of the worker in the alternative match, the choice of employer takes into account the accumulation of human capital and is not based on a simple comparison of the worker's expected product in each of the two competing matches.

In the second or general model, the worker can undertake one of two tasks in each firm, and the current match results in learning not only about the worker's aptitude for the two tasks that she can undertake for the current employer, but also about her aptitude for the two tasks that she can undertake for the alternative employer. As a result, all three components of the wage are nonzero. More important, provided that learning about the quality of the alternative match takes place more slowly in the current match than it would in the alternative match itself, the wage is a convex combination of the worker's static and dynamic outside options. Since better information about the worker's aptitude results in an improvement in both options, there is a positive return to tenure.

Section II contains a brief discussion of related literature. Section III presents the basic model. Efficiency, existence, and uniqueness are established for this model in Section IV, and wage and turnover dynamics are examined in Section V. Section VI presents and analyzes the general model. Section VII presents concluding remarks.

II. Related Literature

The papers most relevant to the current paper would appear to be Jovanovic (1979), Burdett and Mortensen (1989), Bertola and Felli (1993), Bolton and Harris (1993), and Bergemann and Valimaki (1994).

Jovanovic (1979) considers a model in which an infinite number of firms compete for the services of a single worker. The productivities of the matches are independent, identically distributed, and normal ex ante, and they remain independent and normal over time. The worker is assumed to incur a lump-sum cost whenever she switches jobs. Finally, Jovanovic concentrates on an equilibrium in which the worker is paid the expectation—conditional on current information—of her product in the current match at all times.

The wage is not constant through tenure in Jovanovic's model because of the constant arrival of new information about the quality of the current match. It is, however, constant on average because the worker's expected product follows a martingale. According to the narrow sense of return to tenure adopted in the present paper, then, there is no return to tenure in Jovanovic's model.[2] The wage also changes discontinuously on match termination, and it may go either up or down. Upward jumps in the wage are, however, attributable entirely to the cost of switching jobs: in the limiting case with zero costs, the wage can only fall on match termination. Downward jumps in the wage are attributable entirely to option value: the variance of the estimate of the value of the new match is higher, and the new match can always be abandoned if it proves to be unsatisfactory.[3] Our basic model therefore differs from that of Jovanovic in that we obtain upward jumps in the wage without a switching cost, and our general model differs from his in that we do obtain a return to tenure.

The wage-setting rule used by Jovanovic can be rationalized in at least two ways. It is *a* subgame-perfect equilibrium of an extensive form in which, in each period, (i) firms simultaneously and independently propose long-term contracts to the worker and (ii) the worker either chooses one of the contracts offered or chooses to continue in the contract with the current employer.[4] It is also *a* subgame-perfect equilibrium of the extensive form we adopt in the present paper. It cannot, however, be rationalized as a trembling-hand-perfect equilibrium of either extensive form if firms are not identical ex ante: if the worker's prospects are better in one firm than in the others, then that firm will be able to capture some of the surplus, even when firms offer long-term contracts.[5] Our analysis differs from that of Jovanovic in that we are completely explicit about the extensive form we use and our wage-setting rule is the unique trembling-hand-perfect equilib-

[2] There is a return to tenure in Jovanovic's model in the broader sense that the average wage of a cohort of workers, all of whom begin work at the same time, increases on average over time. The same is true of both our models.

[3] In our model, both upward and downward jumps are attributable to the differential between the willingness of the two firms to pay for further investments in specific capital. The willingness to pay for specific capital—i.e., information—is, of course, closely linked to the option of acting on that information later.

[4] Jovanovic's own justification of his wage-setting rule is close in spirit to the one described here.

[5] Jovanovic (1984) himself notes that the contracts he uses are not time consistent.

rium of the bargaining that takes place in the context of our extensive form.[6]

Being explicit about the extensive form that governs the bargaining among the market participants has at least three advantages. First, while choosing the extensive form does involve an implicit assumption as to how bargaining power is distributed among the market participants, the choice of extensive form is more basic than the choice of bargaining power. Second, an extensive form can be compared directly with actual labor market institutions. Third, being explicit about the extensive form focuses attention on how dynamic considerations enter into the bargaining among the market participants.

Two more minor differences between our work and that of Jovanovic (1979) are that (i) we assume the worker's product in a firm to be binomially rather than normally distributed and (ii) we consider the case of two rather than an infinite number of firms. The main advantage of the binomial assumption is that it allows us to depict the dynamics of the basic model using a two-dimensional phase diagram. Indeed, the vast bulk of our analysis carries over more or less directly to the normal case.[7] The main difference that results from the two-firm assumption is that the worker's outside option becomes fully endogenous, falling over time as matches are terminated.

Bertola and Felli (1993) (see also Felli and Harris 1996) consider a model that is the same as our basic model except that they allow for an arbitrary finite number of firms, the worker's product in each firm is normally distributed, and the worker is myopic. They note the interest of adapting their model to the case in which the consumer is not myopic and suggest that it might be possible to make progress with the adapted model under suitable simplifying assumptions. The present paper takes up this suggestion.

Bolton and Harris (1993) consider a rather different learning environment, namely a many-player, common-value extension of the clas-

[6] Equilibria that are subgame perfect but not trembling-hand perfect are not robust: in such equilibria, one or more firm must be bidding more than the value it places on the worker.

[7] In the normal versions of both our models, equilibrium exists and is efficient, equilibrium payoffs are unique, and the worker is paid the full value of the alternative match. In the normal version of our basic model, the worker is paid her static outside option, the wage is constant through tenure, the wage changes discontinuously on match termination, and the wage can go either up or down on match termination. In the normal version of the general model, the wage is a convex combination of the worker's static and dynamic outside options, and there is a positive return to tenure. The main sacrifice in moving to the normal case—other than the phase diagram—is that it is no longer possible to obtain analytic formulae for the equilibrium payoffs of the players and the equilibrium choice of employer.

sic two-armed bandit problem. The modeling techniques of the present paper were, however, adapted from their paper. In particular, like them, we work with the binomial assumption.

Bergemann and Valimaki (1994) consider a model that is closely related to our basic model. They show that all subgame-perfect equilibria of their model—in which each of two sellers attempts to sell the experience good he produces to a single consumer—are efficient, and they highlight one particular subgame-perfect equilibrium, which they dub "cautious," in which the successful seller charges the expected value to the consumer of the good offered by the alternative seller. These results are established under distributional assumptions about the value of the two goods that are much weaker than the assumptions we make about the productivity of the two matches in our model.[8] Their focus on experience goods then leads them to a detailed analysis of the way in which the distribution of the social surplus varies over the set of all possible subgame-perfect equilibria. It should also be noted that they consider a discrete-time version of the underlying learning process, whereas we work in continuous time.

Finally, we would like to mention that, in specifying the extensive form of the wage negotiation, our analysis is related to Burdett and Mortensen (1989). They highlight the importance of specifying the extensive form in models in which matches are an inspection rather than an experience good.

III. The Basic Model

There are two firms, indexed by $i \in \{1, 2\}$, and one worker. For concreteness, we think of firm 1 as a research university, of firm 2 as a teaching university, and of the worker as an academic.[9] The worker may or may not have an aptitude for research. If she does, then her productivity μ_1 in firm 1 takes the value H_1. If she does not, then μ_1 takes the value L_1. Similarly, the worker may or may not have an aptitude for teaching. If she does, then $\mu_2 = H_2$. If she does not, then $\mu_2 = L_2$. Here $H_i \geq L_i$ by convention, and in order to avoid trivialities, we assume that $H_1 > L_2$ and $H_2 > L_1$.

At each time t, the two firms simultaneously and independently make wage offers. We denote the wage offer of firm i by w_i. The worker then chooses to work for one or the other firm. If the worker chooses to work for firm i, then firm i produces $dy_i = \mu_i dt + \sigma dW_i$ and the other firm, which we denote by $-i$, produces $dy_{-i} = 0$. Here

[8] They do, however, assume that the values of the two goods are independent.

[9] It should be noted that, although the academic labor market is probably the labor market most familiar to the reader, it has some features that make it quite different from any other labor market (cf. Ransom 1993).

$\mu_i dt$ is the deterministic but unknown contribution to production that results from the worker's productivity μ_i, σ is a strictly positive scalar, and dW_i is a random shock. More precisely, dW_i is the increment, at time t, of a standard Wiener process W_i. Firm i then receives monetary payoff $d\pi_i = dy_i - w_i dt$, firm $-i$ receives monetary payoff $d\pi_{-i} = 0$, and the worker receives the monetary payoff $d\eta = w_i dt$. Finally, the entire cycle recommences. It should be stressed that, at each stage in the cycle, all market participants are informed of the outcomes of all previous stages of the cycle.

We shall assume that when the worker enters the labor market for the very first time, all market participants hold the same prior beliefs as to the joint distribution of μ_1, μ_2, W_1, and W_2. Since all market participants obtain the same information in the course of the game, it follows that they will hold the same posterior beliefs as to the joint distribution of μ_1, μ_2, W_1, and W_2 as well. Our model therefore has symmetric incomplete information. In particular, no issues of moral hazard or adverse selection arise.

We shall also assume that all market participants believe initially that μ_1, μ_2, W_1, and W_2 are independent. The assumption that μ_1 and μ_2 are independent means that the worker's aptitudes for research and for teaching are, a priori, independent. This is an extreme assumption, but it is in keeping with our desire to develop a theory of firm-specific informational human capital. The assumption that the pair (μ_1, W_1) is independent of the pair (μ_2, W_2) means that the market participants learn nothing about the worker's teaching ability by observing her research performance and nothing about her research ability by observing her teaching performance. Once again, this is an extreme assumption. It is in keeping with our desire to develop a theory of firm-specific informational human capital. Finally, the assumption that μ_i is independent of W_i ensures that while market participants acquire some information about the worker's productivity in firm i by observing her production in firm i, the information obtained is noisy.

Suppose that market participants believe initially that $\mu_i = L_i$ with probability $1 - p_i$ and $\mu_i = H_i$ with probability p_i. Production flows then provide additional information. This information leads market participants to update the p_i using Bayes's rule.[10] We denote the change in p_i by dp_i. It can be shown that if the worker works for firm

[10] In the models of Jovanovic (1979, 1984), market participants believe initially that μ_i is normally distributed. Their beliefs may therefore be summarized by two quantities: the expectation of the productivity of the worker and the variance of the productivity of the worker. Observation of production flows then leads them to revise both these quantities. The expectation evolves stochastically. The variance declines deterministically.

i, then dp_i has mean zero and variance $\Sigma_i(p_i)\,dt$, where

$$\Sigma_i(p_i) = \left[\frac{p_i(1 - p_i)(H_i - L_i)}{\sigma}\right]^2$$

and $dp_{-i} = 0$.

Several points should be noted about this updating rule. First, since the match between the worker and firm i lasts only for an infinitesimal amount of time, only an infinitesimal amount of information is acquired in the course of the match. That is why the change in p_i is infinitesimal. Second, as one would expect, p_i follows a martingale. That is why the mean of dp_i is zero. Third, p_i can never leave the interval [0, 1]. Indeed, if $p_i \in (0, 1)$, then dp_i is infinitesimal. If, on the other hand, $p_i \in \{0, 1\}$, then the variance of dp_i is zero. Hence dp_i is zero. Either way, p_i must remain within the interval [0, 1]. Fourth, $\Sigma_i(p_i)$ is the appropriate measure of the quantity of information revealed by the match: the more the information, the greater the impact on beliefs. In particular, $\Sigma_i(p_i) = 0$ if either (i) $p_i \in \{0, 1\}$, in which case both the firm and the worker are already certain about the quality of the match and so do not revise their beliefs, or (ii) $H_i = L_i$, in which case there is really nothing to learn. Finally, only one of the two beliefs p_1 and p_2 changes at any given time.

The beliefs p_1 and p_2 are the natural state variables of the problem. Their evolution summarizes the market participants' learning about the productivities of the worker. Throughout the paper we restrict attention to stationary Markov strategies for both the firms and the worker. That is, we assume that firms' wage offers W_i are measurable mappings from the state space into the real line $\mathbb{R}$, whereas the worker's choice of employer ι is a measurable mapping from the vector of state variables and wage offers into the set of possible employers {1, 2}: ω_i: $[0, 1]^2 \rightarrow \mathbb{R}$ and ι: $[0, 1]^2 \times \mathbb{R}^2 \rightarrow \{1, 2\}$.

IV. Equilibrium

According to the principles of dynamic programming, U_1, U_2, and V are the equilibrium value functions of firm 1, firm 2, and the worker, respectively, if and only if, for all $(p_1, p_2) \in [0, 1]^2$, $U_1(p_1, p_2)$, $U_2(p_1, p_2)$, and $V(p_1, p_2)$ are the payoffs of firm 1, firm 2, and the worker in an equilibrium of the stage game when the current state is (p_1, p_2) and continuation payoffs are given by U_1, U_2, and V.[11]

[11] This result can take one of two forms: U_1, U_2, and V are the value functions of a subgame-perfect (respectively trembling-hand-perfect) equilibrium in stationary Markov strategies of the dynamic game if and only if, for all $(p_1, p_2) \in [0, 1]^2$, $U_1(p_1, p_2)$, $U_2(p_1, p_2)$, and $V(p_1, p_2)$ are the payoffs of firm 1, firm 2, and the worker

A. *Analysis of Equilibrium in the Stage Game*

In order to characterize equilibrium value functions, then, we must begin by finding equilibrium payoffs in the stage game. Let $m_i = (1 - p_i)L_i + p_iH_i$ denote the expected productivity of the worker in firm i based on current information, and use $\partial_i = \partial/\partial p_i$ and $\partial_i^2 = \partial^2/\partial p_i^2$ for notational convenience. Then we have the following lemma.

LEMMA 1. Suppose that the continuation payoffs of the three market participants are given by U_1, U_2, and V; that firms offer wages w_1 and w_2; and that the worker chooses to work for firm i. Then the expected payoffs of firm i, firm $-i$, and the worker from the stage game are

$$U_i + \left(m_i - w_i + \frac{1}{r}\frac{\partial_i^2 U_i}{2}\Sigma_i\right)dt, \tag{1}$$

$$U_{-i} + \left(\frac{1}{r}\frac{\partial_i^2 U_{-i}}{2}\Sigma_i\right)dt, \tag{2}$$

and

$$V + \left(w_i + \frac{1}{r}\frac{\partial_i^2 V}{2}\Sigma_i\right)dt, \tag{3}$$

respectively.

Here m_i is the expected product if firm i employs the worker; w_i is the wage paid; $1/r$ is the discount factor to be applied to future benefits; $\partial_i^2 U_i/2$, $\partial_i^2 U_{-i}/2$, and $\partial_i^2 V/2$ are the values placed by firm i, firm $-i$, and the worker, respectively, on information about the productivity of the worker in firm i; and Σ_i is the quantity of information about the worker's productivity in firm i revealed by the match.

Notice that the payoffs (1)–(3) are not equilibrium payoffs; we have suppressed the dependence of m_i and Σ_i on p_i and the dependence of U_1, U_2, and V on (p_1, p_2); and there is a close analogy between the value of information to the market participants and the usual Arrow-Pratt measure of local risk aversion.

Proof. The current payoff of firm i is $[m_i(p_i) - w_i]dt$, and its continuation payoff is

$$e^{-rdt}U_i(\mathbf{p} + \mathbf{dp}) = (1 - rdt)[U_i(\mathbf{p}) + \mathbf{dp}\cdot\boldsymbol{\partial}\mathbf{U}_i(\mathbf{p}) + \tfrac{1}{2}\mathbf{dp}\cdot\boldsymbol{\partial}^2\mathbf{U}_i(\mathbf{p})\,\mathbf{dp}],$$

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in a subgame-perfect (trembling-hand-perfect) equilibrium of the stage game when continuation payoffs are given by U_1, U_2, and V. It is the trembling-hand-perfect formulation that we need. For the result to hold in this formulation, it is necessary to specify that the trembles are themselves Markovian.

where $\mathbf{p} = \binom{p_1}{p_2}$, $\mathbf{dp} = \binom{dp_1}{dp_2}$, $\boldsymbol{\partial}\mathbf{U}_i$ denotes the vector of first derivatives of U_i with respect to $\mathbf{p}$, and $\boldsymbol{\partial}^2\mathbf{U}_i$ denotes the matrix of second derivatives of U_i with respect to $\mathbf{p}$. Now dp_i has mean zero and variance $\Sigma_i(p_i)dt$ and dp_{-i} is identically zero. Hence the expected payoff of firm i from the stage game is

$$[m_i(p_i) - w_i]dt + (1 - rdt)\left[U_i(\mathbf{p}) + \frac{\partial_i^2 U_i(\mathbf{p})}{2}\Sigma_i(p_i)dt\right].$$

Dropping terms of order dt^2 and rearranging yields expression (1). Expressions (2) and (3) can be derived similarly. Q.E.D.

Armed with lemma 1, we obtain the following theorem.

THEOREM 1. Suppose once again that the continuation payoffs of the market participants are given by U_1, U_2, and V. Then the stage game possesses a unique equilibrium.[12] In this equilibrium, the worker's choice of employer is

$$I \in \underset{i}{\operatorname{argmax}}\left\{m_i + \frac{1}{r}\frac{\partial_i^2 V}{2}\Sigma_i + \frac{1}{r}\frac{\partial_i^2 U_i}{2}\Sigma_i - \frac{1}{r}\frac{\partial_{-i}^2 U_i}{2}\Sigma_{-i}\right\}, \quad (4)$$

and she receives wage

$$W = m_{-I} + \frac{1}{r}\left(\frac{\partial_{-I}^2 V}{2}\Sigma_{-I} + \frac{\partial_{-I}^2 U_{-I}}{2}\Sigma_{-I} - \frac{\partial_I^2 V}{2}\Sigma_I - \frac{\partial_I^2 U_{-I}}{2}\Sigma_I\right). \quad (5)$$

[12] More precisely, the stage game possesses a continuum of subgame-perfect equilibria and a unique trembling-hand-perfect equilibrium. Indeed, consider the case in which, in the notation of the proof of the theorem,

$$B_i + \frac{1}{r}\frac{\partial_i^2 V}{2}\Sigma_i > B_{-i} + \frac{1}{r}\frac{\partial_{-i}^2 V}{2}\Sigma_{-i}.$$

In this case, for each wage W such that

$$B_{-i} + \frac{1}{r}\frac{\partial_{-i}^2 V}{2}\Sigma_{-i} - \frac{1}{r}\frac{\partial_i^2 V}{2}\Sigma_i \leq W \leq B_i,$$

there is a subgame-perfect equilibrium of the stage game in which the worker works for firm i and is paid the wage W. However, only the subgame-perfect equilibrium in which

$$W = B_{-i} + \frac{1}{r}\frac{\partial_{-i}^2 V}{2}\Sigma_{-i} - \frac{1}{r}\frac{\partial_i^2 V}{2}\Sigma_i$$

is trembling-hand perfect. The reason is that a wage

$$W > B_{-i} + \frac{1}{r}\frac{\partial_{-i}^2 V}{2}\Sigma_{-i} - \frac{1}{r}\frac{\partial_i^2 V}{2}\Sigma_i$$

can be sustained only if the alternative employer $-i$ bids more than its true willingness to pay for the services of the worker. Such a bid runs the risk that it might be taken up.

In order to interpret the wage, it will be helpful to introduce the quantities $S_i = U_i + V$, which are the equilibrium payoffs of the pairwise coalition consisting of firm i and the worker. In terms of these quantities, the wage can be written in the form

$$W = m_{-I} + \frac{1}{r}\frac{\partial_{-I}^2 S_{-I}}{2}\Sigma_{-I} - \frac{1}{r}\frac{\partial_I^2 S_{-I}}{2}\Sigma_I. \tag{6}$$

In other words, the equilibrium wage is the full value of the benefit to the coalition consisting of the worker and firm $-I$ of persuading the worker to switch from firm I to firm $-I$. This benefit is made up of three components: the expected product m_{-I} of the worker in firm $-I$, the value $(1/r)(\partial_{-I}^2 S_{-I}/2)\Sigma_{-I}$ of the information that would be gained by the coalition consisting of the worker and firm $-I$ if the worker were to switch, and the value $(1/r)(\partial_I^2 S_{-I}/2)\Sigma_I$ of the information that the same coalition would forgo if the worker were to switch.

The latter two components can be interpreted in terms of our motivating example. Consider first the case of a hotshot full professor. If the teaching university wishes to persuade her to work for it, then it will have to raise her wage above her expected product to compensate her for the human capital accumulation that she forgoes by working for it. Consider now the case of an aspiring assistant professor. If the research university employs her, then it will be able to take advantage of the human capital accumulation that she is able to achieve by working for it to lower her wage below her expected product.

Proof. If firm i succeeds in securing the services of the worker, then it obtains expected payoff

$$U_i + \left(m_i - w_i + \frac{1}{r}\frac{\partial_i^2 U_i}{2}\Sigma_i\right)dt,$$

and if it fails, then it obtains expected payoff

$$U_i + \left(\frac{1}{r}\frac{\partial_{-i}^2 U_i}{2}\Sigma_{-i}\right)dt.$$

Hence it is willing to bid up to

$$B_i = m_i + \frac{1}{r}\frac{\partial_i^2 U_i}{2}\Sigma_i - \frac{1}{r}\frac{\partial_{-i}^2 U_i}{2}\Sigma_{-i}$$

for the services of the worker. On the other hand, if the worker works for firm i, then she receives expected payoff

$$V + \left(w_i + \frac{1}{r}\frac{\partial_i^2 V}{2}\Sigma_i\right)dt.$$

Hence she will work for firm i if and only if

$$w_i + \frac{1}{r}\frac{\partial_i^2 V}{2}\Sigma_i > w_{-i} + \frac{1}{r}\frac{\partial_{-i}^2 V}{2}\Sigma_{-i}.$$

Overall, then, firm i will secure the services of the worker if and only if

$$B_i + \frac{1}{r}\frac{\partial_i^2 V}{2}\Sigma_i > B_{-i} + \frac{1}{r}\frac{\partial_{-i}^2 V}{2}\Sigma_{-i}.$$

If it does secure the services of the worker, it will pay the minimum wage necessary to outbid firm $-i$, namely

$$W = B_{-i} + \frac{1}{r}\frac{\partial_{-i}^2 V}{2}\Sigma_{-i} - \frac{1}{r}\frac{\partial_i^2 V}{2}\Sigma_i.$$

Q.E.D.

Theorem 1 leads immediately to formulae for the equilibrium payoffs of the stage game.

COROLLARY 1. Suppose that the continuation payoffs of the three market participants are given by U_1, U_2, and V. Then the equilibrium payoffs of firm I, firm $-I \neq I$, and the worker from the stage game are

$$U_I + \left(m_I - W + \frac{1}{r}\frac{\partial_I^2 U_I}{2}\Sigma_I\right)dt, \tag{7}$$

$$U_{-I} + \left(\frac{1}{r}\frac{\partial_I^2 U_{-I}}{2}\Sigma_I\right)dt, \tag{8}$$

and

$$V + \left(W + \frac{1}{r}\frac{\partial_I^2 V}{2}\Sigma_I\right)dt, \tag{9}$$

respectively, where I and W are as in theorem 1. Q.E.D.

B. The Bellman Equations for the Dynamic Game

From (4)–(5) and (7)–(9), it follows that U_1, U_2, and V are the value functions of an equilibrium if and only if they satisfy the system of Bellman equations

$$U_I = m_I - W + \frac{1}{r}\frac{\partial_I^2 U_I}{2}\Sigma_I, \tag{10}$$

$$U_{-I} = \frac{1}{r}\frac{\partial_I^2 U_{-I}}{2}\Sigma_I, \tag{11}$$

and

$$V = W + \frac{1}{r}\frac{\partial_I^2 V}{2}\Sigma_I, \tag{12}$$

where

$$I \in \underset{i}{\operatorname{argmax}}\left\{m_i + \frac{1}{r}\left(\frac{\partial_i^2 V}{2}\Sigma_i + \frac{\partial_i^2 U_i}{2}\Sigma_i - \frac{\partial_{-i}^2 U_i}{2}\Sigma_{-i}\right)\right\} \tag{13}$$

and

$$W = m_{-I} + \frac{1}{r}\left(\frac{\partial_{-I}^2 V}{2}\Sigma_{-I} + \frac{\partial_{-I}^2 U_{-I}}{2}\Sigma_{-I} - \frac{\partial_I^2 V}{2}\Sigma_I - \frac{\partial_I^2 U_{-I}}{2}\Sigma_I\right). \tag{14}$$

C. The Optimal-Retirement Problem

In order to characterize equilibrium in our model, we need to introduce an auxiliary problem. In the new problem, a self-employed worker has access to the technology of firm *i*. She must choose between continuing to work with this technology, and retiring and obtaining a lump-sum payment $R \in [L_i, H_i]$. Let $\phi_i(p_i, R)$ denote the value of this optimal-stopping problem to the worker,[13] and let $\psi_i(p_i) = \sup\{R \mid \phi_i(p_i, R) > R\}$.[14] That is, let $\psi_i(p_i)$ be the maximum value of R at which the worker is willing to continue work when her productivity estimate is p_i. We refer to ψ_i as the dynamic allocation index of firm *i*.

D. Analysis of Equilibrium in the Dynamic Game

Suppose that U_1, U_2, and V are the value functions of an equilibrium of the dynamic game, and let $S = U_1 + U_2 + V$ be the equilibrium value of the grand coalition consisting of both firms and the worker. Then our first result about equilibrium in the dynamic game is the following lemma.

[13] Let $\gamma_i = \sqrt{1 + [8r\sigma^2/(H_i - L_i)^2]}$ and $g_i(p_i) = p_i^{-(\gamma_i - 1)/2}(1 - p_i)^{(\gamma_i + 1)/2}$, and let c_i and λ_i be the unique solution of the pair of equations $m_i(c_i) + \lambda_i g_i(c_i) = R$ and $m_i'(c_i) + \lambda_i g_i'(c_i) = 0$. That is, let $c_i = [(\gamma_i - 1)(R - L_i)]/[2(H_i - R) + (\gamma_i - 1)(H_i - L_i)]$ and $\lambda_i = -m_i'(c_i)/g_i'(c_i)$. Then $\phi_i(p_i, R) = R$ if $p_i \le c_i$, and $\phi_i(p_i, R) = m_i(p_i) + \lambda_i g_i(p_i)$ if $p_i \ge c_i$.

[14] In other words, let $\psi_i(p_i)$ be the smallest R such that $\phi_i(p_i, R) = R$. In the notation of n. 13, we have $\psi_i(p_i) = m_i(p_i) - [m_i'(p_i)g_i(p_i)/g_i'(p_i)]$.

LEMMA 2. S is equal to the value function for the grand-team problem in which the market participants act cooperatively to maximize the sum of their payoffs, and I is an optimal policy for this problem. In particular, any equilibrium of the dynamic game is socially efficient, the equilibrium value of the grand coalition consisting of all three market participants is the same for all equilibria, and this value can be calculated without reference to any equilibrium considerations.[15]

Lemma 2 can be thought of as an instance of the Coase theorem. In our problem, the worker has the right to decide which firm to work for. Her decision has implications for both firms, and she does not take any direct account of them. So an inefficiency could, in principle, arise. Each firm can, however, alter the worker's decision at the margin by making an appropriate adjustment to the wage it offers. As a result, the worker is led to take indirect account of the externalities that she generates for the firms, and the outcome is efficient after all.

It should be stressed that lemma 2 does not generalize to the case of three or more firms.[16] The explanation in terms of the Coase theorem makes clear why this is the case. With three or more firms, a firm can adjust the wage it offers so as to influence the worker's choice between itself and the other firms, but it cannot influence the worker's choice among the other firms.

Finally, note that the grand-team problem is a two-armed bandit problem. For a treatment of the multiarmed bandit problem in continuous time, see Karatzas (1984).

Proof. Adding (10), (11), and (12) and noting that

$$m_i + \frac{1}{r}\frac{\partial_i^2 V}{2}\Sigma_i + \frac{1}{r}\frac{\partial_i^2 U_i}{2}\Sigma_i - \frac{1}{r}\frac{\partial_{-i}^2 U_i}{2}\Sigma_{-i}$$

$$\geq m_{-i} + \frac{1}{r}\frac{\partial_{-i}^2 V}{2}\Sigma_{-i} + \frac{1}{r}\frac{\partial_{-i}^2 U_{-i}}{2}\Sigma_{-i} - \frac{1}{r}\frac{\partial_i^2 U_{-i}}{2}\Sigma_i$$

if and only if

$$m_i + \frac{1}{r}\frac{\partial_i^2 S}{2}\Sigma_i \geq m_{-i} + \frac{1}{r}\frac{\partial_{-i}^2 S}{2}\Sigma_{-i},$$

[15] Efficiency is not confined to trembling-hand-perfect equilibria: it holds for all subgame-perfect equilibria.

[16] Although trembling-hand-perfect equilibria are probably inefficient in general, there do exist efficient subgame-perfect equilibria. For example, there is an equilibrium in which the wage is equal to the worker's expected marginal product with her current employer.

we obtain from (10)–(12) that

$$S = m_I + \frac{1}{r}\frac{\partial_I^2 S}{2}\Sigma_I \tag{15}$$

and from (13) that

$$I \in \operatorname*{argmax}_i \left\{ m_i + \frac{1}{r}\frac{\partial_i^2 S}{2}\Sigma_i \right\}. \tag{16}$$

Finally, combining (15) and (16), we have

$$S = \max_i \left\{ m_i + \frac{1}{r}\frac{\partial_i^2 S}{2}\Sigma_i \right\}. \tag{17}$$

That is, S solves the Bellman equation for the grand-team problem. Since the Bellman equation for the grand-team problem has a unique solution, S must be equal to the value function for this problem. Finally, since S is equal to the value function for the grand-team problem, (16) implies that I is an optimal policy for this problem. Q.E.D.

Our second result concerns the equilibrium value S_i of the pairwise coalition consisting of firm i and the worker.

LEMMA 3. S_i is equal to the value function for the pairwise-team problem in which the worker works permanently for firm i and in which firm i and the worker act cooperatively to maximize the sum of their payoffs.

In particular, the equilibrium value of the pairwise coalition consisting of firm i and the worker is the same for all equilibria, and this value can be calculated without reference to any equilibrium considerations.

Notice that once the worker teams up permanently with firm i, there are no decisions to make. So the pairwise-team problem is very simple.

Proof. Adding (10) and (12), we obtain

$$S_I = m_I + \frac{1}{r}\frac{\partial_I^2 S_I}{2}\Sigma_I,$$

and adding (11) and (12), we obtain

$$S_{-I} = m_{-I} + \frac{1}{r}\frac{\partial_{-I}^2 S_{-I}}{2}\Sigma_{-I}.$$

Combining these two equations, we see that

$$S_i = m_i + \frac{1}{r}\frac{\partial_i^2 S_i}{2}\Sigma_i \tag{18}$$

for $i \in \{1, 2\}$. That is, S_i solves the Bellman equation for the pairwise-team problem for pairwise team i. Q.E.D.

Our third result concerns existence.

LEMMA 4. Equilibrium exists.

Proof. The first step is to find a solution S for the Bellman equation (17) for the grand-team problem. This equation is a weakly nondegenerate, nonlinear elliptic partial differential equation. It can be verified that it possesses the solution

$$S(p_1, p_2) = \max\{H_1, H_2\} - \int_{\min\{L_1,L_2\}}^{\max\{H_1,H_2\}} \left[\frac{\partial\phi_1(p_1, R)}{\partial R}\right]\left[\frac{\partial\phi_2(p_2, R)}{\partial R}\right] dR \tag{19}$$

(cf. Karatzas 1984).[17]

The second step is to find a solution for the Bellman equation (18) for pairwise-team problem i. This equation is a linear elliptic ordinary differential equation. It is easy to check that it possesses the solution $S_i(p_1, p_2) = m_i(p_i)$.

The third step is to use the value functions S and S_i obtained in the first two steps to obtain equilibrium value functions for the market participants. We get $V = S_1 + S_2 - S$, $U_1 = S - S_2$, and $U_2 = S - S_1$.

The fourth step is to find equilibrium strategies for the players. We begin with an optimal policy I for the grand-team problem. Next, we define strategies ω_1 and ω_2 for the two firms by the formula

$$\omega_1 = \omega_2 = m_{-I} + \frac{1}{r}\left(\frac{\partial^2_{-I}V}{2}\Sigma_{-I} + \frac{\partial^2_{-I}U_{-I}}{2}\Sigma_{-I} - \frac{\partial^2_I V}{2}\Sigma_I - \frac{\partial^2_I U_{-I}}{2}\Sigma_I\right)$$

(cf. theorem 1). Finally, we define a strategy ι for the worker by the formula

$$\iota = \begin{cases} 1 & \text{if } w_1 + \frac{1}{r}\frac{\partial_1^2 V}{2}\Sigma_1 > w_2 + \frac{1}{r}\frac{\partial_2^2 V}{2}\Sigma_2 \\ 2 & \text{if } w_2 + \frac{1}{r}\frac{\partial_2^2 V}{2}\Sigma_2 > w_1 + \frac{1}{r}\frac{\partial_1^2 V}{2}\Sigma_1 \\ I & \text{if } w_1 + \frac{1}{r}\frac{\partial_1^2 V}{2}\Sigma_1 = w_2 + \frac{1}{r}\frac{\partial_2^2 V}{2}\Sigma_2 \end{cases}$$

[17] Notice that the work of Karatzas (1984) does not, strictly speaking, apply here. He works with the case in which the state space is unbounded and the variance of the increments dp_i is bounded away from zero. In particular, it is never optimal to abandon an arm altogether in his problem. One implication of the difference between our setting and that of Karatzas is that the function S need not be twice continuously differentiable. Although it is twice continuously differentiable in the interior of the

(cf. theorem 1 again). It is easy to verify that strategies defined in this way constitute an equilibrium. Q.E.D.

Using lemmas 2–4, we obtain the following theorem.

THEOREM 2. Equilibrium exists and is efficient, and equilibrium payoffs are unique.

Notice that we do not claim that equilibrium itself is unique, only that equilibrium payoffs are unique.[18] This appears to be the economically relevant form of uniqueness: it implies that the extensive form that we have chosen to govern the bargaining between the market participants serves to tie down their shares in the social surplus uniquely.

Proof. It remains only to verify that equilibrium payoffs are unique. We have already noted that if U_1, U_2, and V are equilibrium value functions, then $S = U_1 + U_2 + V$, $S_1 = U_1 + V$, and $S_2 = U_2 + V$ are the value functions for the grand-team problem, pairwise-team problem 1, and pairwise-team problem 2. Hence S_1, S_2, and S are unique. In view of the simple identities $V = S_1 + S_2 - S$, $U_1 = S - S_2$, and $U_2 = S - S_1$, the equilibrium value functions U_1, U_2, and V must be unique too. Q.E.D.

V. Wage and Turnover Dynamics

There is a particularly simple characterization of the equilibrium choice of employer by the worker in our model.

A. *The Equilibrium Choice of Employer*

As in Section IV*C*, let $\phi_i(p_i, R)$ be the value of the optimal-retirement problem in which the worker has access to the technology of firm i, and let $\psi_i(p_i)$ be the maximum value of R at which the worker is willing to continue work when her productivity estimate is p_i. Then we have the following theorem.

THEOREM 3. The state space $[0, 1]^2$ divides into three regions: (1) in region 1, which consists of those states $\mathbf{p}$ such that $\psi_1(p_1) > \psi_2(p_2)$, the worker works for firm 1; (2) in region 2, which consists of those states $\mathbf{p}$ such that $\psi_1(p_1) < \psi_2(p_2)$, the worker works for firm 2; and (3) in region 3, which consists of those states $\mathbf{p}$ such that $\psi_1(p_1) = \psi_2(p_2)$, the worker is indifferent as to her choice of employer.

state space, it may have discontinuities in its second derivatives at the intersection between the boundary and the line of equal index. (The line of equal index is the locus of points $\mathbf{p}$ such that $\psi_1(p_1) = \psi_2(p_2)$.)

[18] It can also be shown that the equilibrium evolution of the state—which is a probability distribution over the set of continuous mappings from $[0, \infty)$ to $[0, 1]^2$—is unique. The proof builds on Karatzas's (1984) analysis of uniqueness for the two-armed case.

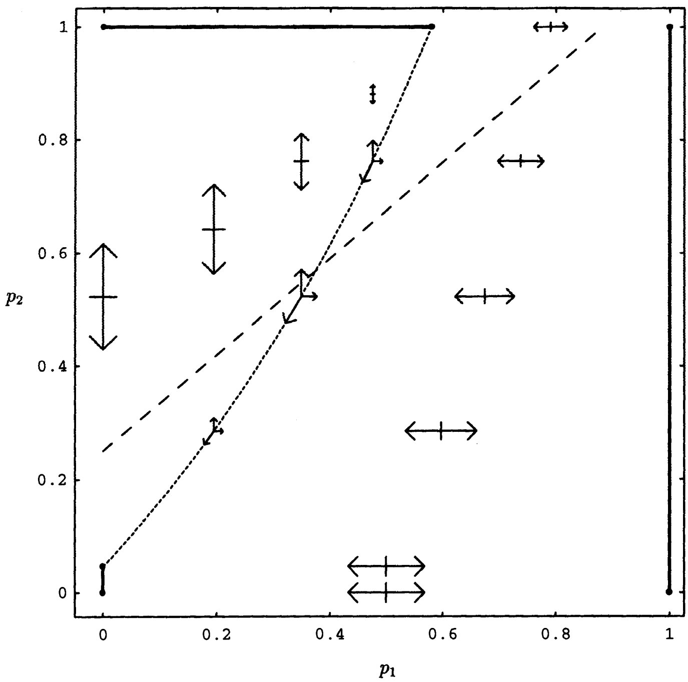

FIG. 1.—Phase diagram of the basic model

All three of these regions have economic significance. To understand what they look like, note that $\psi_i(0) = L_i$, ψ_i is strictly increasing on the interval [0, 1], and $\psi_i(1) = H_i$. Hence region 3 is an upward-sloping curve that begins at some point on the lower or left-hand boundary of the state space[19] and ends at some point on the upper or right-hand boundary of the state space.[20] Region 1 is the region *below* this line, and region 2 is the region *above* this line.

If the state is in region 1, then $dp_2 = 0$, and it therefore evolves horizontally and stochastically. If the state is in region 2, then $dp_1 = 0$, and it therefore evolves vertically and stochastically. Finally, its dynamics in region 3 are more subtle. In effect, it moves monotonically down and left within the region. This movement is, however, constantly interrupted by random excursions into regions 1 and 2.

Figure 1 depicts the state space in the case in which $L_1 = 0.25$, $H_1 = 1.1$, $L_2 = 0$, $H_2 = 1$, $r = 0.1$, and $\sigma = 1$. For these parameter values, $H_1 > H_2$ and $L_1 > L_2$. Hence the locus of equal product (i.e.,

[19] That is, at some (p_1, p_2) such that $\min\{p_1, p_2\} = 0$ and $\max\{p_1, p_2\} < 1$.
[20] That is, at some (p_1, p_2) such that $\min\{p_1, p_2\} > 0$ and $\max\{p_1, p_2\} = 1$.

the set of states **p** such that $m_1(p_1) = m_2(p_2)$) and the locus of equal index (i.e., the set of states **p** such that $\psi_1(p_1) = \psi_2(p_2)$) both lie above the leading diagonal. The locus of equal product is represented by the dashed line, and the locus of equal index is represented by the dotted line. The left-hand end of the locus of equal product lies above the left-hand end of the locus of equal index because the market values information as to whether $\mu_2 = L_2 < L_1$ or $\mu_2 = H_2 > L_1$ when $p_1 = 0$, that is, when it is certain that $\mu_1 = L_1$. Similarly, the right-hand end of the locus of equal product lies to the right of the right-hand end of the locus of equal index because the market values information as to whether $\mu_1 = L_1 < H_2$ or $\mu_1 = H_1 > H_2$ when $p_2 = 1$, that is, when it is certain that $\mu_2 = H_2$.

In region 1, that is, in the region below the dotted line, the state moves right and left with equal probability. The evolution of the state in that region is therefore represented by a horizontal two-headed arrow, the length of the shafts of which is proportional to the size of the movement weighted by the probability that the movement takes place in the direction indicated. Similarly, the evolution of the state in region 2, that is, in the region above the dotted line, is represented by a vertical two-headed arrow. In region 3, the state can exit horizontally into region 1, exit vertically into region 2, or move down and to the left without leaving the region. The evolution of the state in region 3 is therefore represented by a three-headed arrow, the length of the shafts of which is proportional to the size of the movement weighted by the probability that the movement takes place in the direction indicated.

The state does not evolve at all if it lies on the left-hand boundary below the line of equal index, on the upper boundary to the left of the line of equal index, or on the right-hand boundary. The locus of resting points is represented by the solid line. The state must converge to a point of this locus as $t \to \infty$.

Proof. It can be checked that a policy I satisfies the optimality conditions (16) for the grand-team problem if and only if $I \in \operatorname{argmax}_i\{\psi_i\}$. Q.E.D.

B. *Wages*

In order to appreciate the full significance of the three regions of theorem 3, we need a formula for the wage.

THEOREM 4. $W = m_{-I}$.

In other words, the worker is paid her static outside option.

Proof. Formula (6) tells us that

$$W = m_{-I} + \frac{1}{r}\frac{\partial_{-I}^2 S_{-I}}{2}\Sigma_{-I} - \frac{1}{r}\frac{\partial_I^2 S_{-I}}{2}\Sigma_I,$$

the proof of theorem 2 tells us that $S_i = m_i$, and m_i is linear in p_i and independent of p_{-i}. Hence $W = m_{-I}$, as required. Q.E.D.

Theorems 3 and 4 allow us to give a detailed picture of the wage dynamics of our model.

Notice first that, as long as the state $\mathbf{p}$ lies in region $i \in \{1, 2\}$, we have $W = m_{-i}(p_{-i})$ and $dp_{-i} = 0$. So the wage is constant through tenure.

Second, while the wage is determined by the expected products m_i, the choice of employer is determined by the indices ψ_i. Moreover, it can be shown that the locus of equal product coincides with the locus of equal index if and only if the two firms are perfectly symmetric. It follows that the worker may be paid more or less than her expected product in her current employment, the wage will in general change discontinuously on match termination, and this change may be either positive or negative.

Third, in the symmetric case, the dynamic allocation index ψ_i coincides with the expected product m_i. Hence $W = \min\{m_1, m_2\}$. In particular, W is a concave function of $\mathbf{p}$. Now $\mathbf{p}$ follows a martingale. Hence W follows a supermartingale. That is, W falls on average over time.[21] In particular, W falls on average on match termination. It would be interesting to know whether this result carries over to the general case.

Fourth, it is entirely possible that the worker will work for the firm in which her expected product is lower. This occurs when the market values the information about her productivity in firm I more than the extra output that would be obtained if she were to work for firm $-I$.

C. *Turnover*

There are several possible paths that the career of the worker can follow, depending on the initial beliefs $\mathbf{p}(0)$. We shall not attempt to give an exhaustive description of them. We shall concentrate instead on a particular case.

Suppose that $\psi_1(p_1(0)) > \psi_2(p_2(0)) > \psi_1(0)$. The first of these inequalities implies that the worker is initially employed by firm 1. The second implies that she will switch to firm 2 if the market's estimate of her productivity in firm 1 falls far enough. There are then four basic possibilities. All four of these possibilities occur with positive probability.

First, the worker may remain with firm 1 throughout her career.

[21] We can actually show more in the symmetric case: W falls *monotonically* with time, and not just on average.

In this case, p_1 converges to one as $t \to \infty$. In other words, it eventually becomes certain that her productivity in firm 1 is high. Second, she may quit firm 1 at some point and then, following a sequence of turnover events, spend the rest of her career at firm 1. In this case, p_1 again converges to one as $t \to \infty$. In other words, it eventually becomes certain that her productivity in firm 1 is high. Third, she may quit firm 1 at some point and then, following a sequence of turnover events, spend the rest of her career at firm 2. In this case, p_2 converges to one as $t \to \infty$. In other words, it eventually becomes certain that her productivity in firm 2 is high. Fourth, she may quit firm 1 at some point, but the turnover events may continue indefinitely. In this case both $\psi_1(p_1)$ and $\psi_2(p_2)$ converge to $\max\{L_1, L_2\}$. In other words, the productivity estimates converge to the point at which the market is indifferent beween (i) the "risky" option of employing the worker in the firm for which L_i is lower and learning more about her productivity there and (ii) the "safe" option of employing her at her minimum productivity in the firm for which L_i is higher. In particular, it eventually becomes certain that her productivity in the firm for which L_i is higher is low. Finally, it can be shown that she spends an increasingly large proportion of her time working for the firm for which L_i is higher as time goes on.

VI. The General Model

The basic model of Section III had a theoretical limitation, that only the first of the three potential components of the wage was nonzero, and an empirical limitation, that the wage remained constant through tenure. In the present section we introduce a more general model that is free from both these limitations.

In the new model there are two firms and one worker as before. The worker may, however, undertake one of two tasks in each firm: she may undertake economics research or fine-art research in firm 1 and economics teaching or fine-art teaching in firm 2. She has an aptitude for exactly one of the two tasks in each firm. If she has an aptitude for economics in firm i, then her productivity μ_i in that firm takes the value (H_{iE}, L_{iF}). If she has an aptitude for fine art, then μ_i takes the value (L_{iE}, H_{iF}).

Also, research is no longer necessarily exclusive to firm 1, and teaching is no longer necessarily exclusive to firm 2. We assume, rather, that when engaging in the subject economics (fine art) in firm 1, the worker must devote a fraction α_1 of her time to economics research (fine-art research) and a fraction $1 - \alpha_1$ to economics teaching (fine-art teaching). Similarly, when engaging in the subject economics (fine art) in firm 2, the worker must devote a fraction α_2 of her time to economics teaching (fine-art teaching) and a fraction $1 - \alpha_2$ to economics research (fine-art research).

The assumption that the worker has an aptitude for exactly one task in each firm implies that a firm will switch the worker between the two tasks as it learns more about her aptitude. Because it can thereby exploit the informational human capital that she accumulates, this capital comes to have value within the context of the match. The current employer is therefore forced to compensate the worker for the human capital accumulation specific to the alternative match she forgoes by participating in the current match, and the second component of the wage is no longer zero.

The assumption that the worker must devote a fraction of her time in firm 1 to the activity specific to firm 2, namely teaching, and a fraction of her time in firm 2 to the activity specific to firm 1, namely research, implies that some learning concerning her aptitude for the activity associated with each employer takes place even while working for the other. As a result, the current employer is able to reduce the wage to compensate for the human capital accumulation specific to the alternative match she obtains by participating in the current match, and the third component of the wage is no longer zero either.

The combination of the two assumptions implies that the worker's outside option rises with time. Indeed, informational human capital specific to the alternative match has a value, and such capital accumulates in the course of the current match. This is not quite enough to generate returns to tenure, however: one must also ensure that the rate of accumulation of capital specific to firm $-i$ is slower in firm i than it is in firm $-i$.

More precisely, we assume that each firm may assign the worker to one of two tasks $k \in \{E, F\}$. If the worker undertakes task k in firm 1, then she generates the *pair* of outputs

$$\mathbf{dy} = \begin{pmatrix} dy_1 \\ dy_2 \end{pmatrix} = \begin{pmatrix} \alpha_1 \mu_{1k} dt + \sigma\sqrt{\alpha_1}\, dW_1 \\ (1 - \alpha_1)\mu_{2k} dt + \sigma\sqrt{1 - \alpha_1}\, dW_2 \end{pmatrix}, \tag{20}$$

firm 1 receives the monetary payoff $d\pi_1 = dy_1 - w_1 dt$, firm 2 receives the monetary payoff $d\pi_2 = 0$, and the worker receives the monetary payoff $d\eta = w_1 dt$. Similarly, if she works in firm 2, then she generates the *pair* of outputs

$$\mathbf{dy} = \begin{pmatrix} dy_1 \\ dy_2 \end{pmatrix} = \begin{pmatrix} (1 - \alpha_2)\mu_{1k} dt + \sigma\sqrt{1 - \alpha_2}\, dW_1 \\ \alpha_2 \mu_{2k} dt + \sigma\sqrt{\alpha_2}\, dW_2 \end{pmatrix}, \tag{21}$$

firm 1 receives the monetary payoff $d\pi_1 = 0$, firm 2 receives the monetary payoff $d\pi_2 = dy_2 - w_2 dt$, and the worker receives the monetary payoff $d\eta = w_2 dt$. In either case, *both* firms observe *both* components of the vector $\mathbf{dy}$.

We also assume that $H_{ik} > L_{ik}$, that is, that the worker's productivity when she undertakes task k in firm i is higher if she has an aptitude for that task; that $H_{iE} - L_{iE} = H_{iF} - L_{iF}$, that is, that learning about the worker's aptitude in firm i takes place at the same rate in both tasks; that $H_{iE} > H_{iF}$, that is, that the productivity of a worker with an aptitude for economics is higher than the productivity of a worker with an aptitude for fine art; and that $\alpha_i > 0$, that is, that the worker generates at least some output of value to her current employer.

Notice that when firm i employs the worker, it derives no benefit from the time that she devotes to the activity specific to the other firm. The worker must, nonetheless, devote a fraction $1 - \alpha_i$ of her time to that activity. This assumption is crucial if we wish to ensure that all human capital is firm specific.

Notice finally that the basic model is the special case of the present model in which $H_{iF} \leq L_{iE}$ (i.e., fine art is dominated by economics) and $\alpha_i = 1$ (i.e., there are no informational spillovers).

A. *The Updating of Beliefs*

Suppose that the worker works for firm 1. Since she undertakes both research and teaching, both p_1 and p_2 will change following observation of her performance. We must therefore consider the joint distribution of dp_1 and dp_2. Because μ_1, μ_2, W_1, and W_2 are independent,

$$\mathbf{dp} = \begin{pmatrix} dp_1 \\ dp_2 \end{pmatrix}$$

has mean zero and covariance $\tilde{\boldsymbol{\Sigma}}_1(p_1, p_2)dt$, where

$$\tilde{\boldsymbol{\Sigma}}_1(p_1, p_2) = \begin{pmatrix} \alpha_1 \Sigma_1(p_1) & 0 \\ 0 & (1 - \alpha_1)\Sigma_2(p_2) \end{pmatrix}$$

and

$$\Sigma_i(p_i) = \left[\frac{(H_i - L_i)p_i(1 - p_i)}{\sigma}\right]^2$$

as usual. Similarly, if the worker works for firm 2, then $\mathbf{dp}$ has mean zero and covariance $\tilde{\boldsymbol{\Sigma}}_2(p_1, p_2)dt$, where

$$\tilde{\boldsymbol{\Sigma}}_2(p_1, p_2) = \begin{pmatrix} (1 - \alpha_2)\Sigma_1(p_1) & 0 \\ 0 & \alpha_2 \Sigma_2(p_2) \end{pmatrix}.$$

These formulae imply, in particular, that the scaling of the noise in (20) and (21) above is correct: when the worker works for firm i, she produces information $\alpha_i \Sigma_i dt$ concerning the productivity of the match with firm i and information $(1 - \alpha_i)\Sigma_{-i}dt$ concerning the productivity of the match with firm $-i$.

B. *Marginal Products*

If the worker undertakes task k in firm i, then her expected product is $m_{ik}(p_i) = \alpha_i[(1 - p_i)L_{ik} + p_i H_{ik}]$. Since both tasks generate the same amount of information about her aptitude for the activity in which firm i specializes, the firm will assign her to whichever of the two tasks has the higher expected product, and her overall expected product will therefore be $m_i = \max\{m_{iE}, m_{iF}\}$.

There are two basic possibilities for m_i. If $H_{iF} \leq L_{iE}$, then fine art is dominated by economics in firm i and m_i is unchanged from the basic model. If, on the other hand, $H_{iF} > L_{iE}$, then fine art is preferable to economics when the worker's aptitude for economics is low. Hence m_i coincides with the decreasing linear function m_{iF} on the interval $[0, b_i]$ and with the increasing linear function m_{iE} on the interval $[b_i, 1]$, where $b_i = (H_{iF} - L_{iE})/[(H_{iF} - L_{iE}) + (H_{iE} - L_{iF})]$. In particular, m_i is convex.

C. *The Bellman Equations*

As in Section IV, it can be shown that U_1, U_2, and V are the value functions of an equilibrium if and only if they satisfy the system of Bellman equations

$$U_I = m_I - W + \frac{1}{r}\mathrm{Tr}\left[\frac{\partial^2 \mathbf{U}_I}{2}\tilde{\mathbf{\Sigma}}_I\right], \tag{22}$$

$$U_{-I} = \frac{1}{r}\mathrm{Tr}\left[\frac{\partial^2 \mathbf{U}_{-I}}{2}\tilde{\mathbf{\Sigma}}_I\right], \tag{23}$$

and

$$V = W + \frac{1}{r}\mathrm{Tr}\left[\frac{\partial^2 \mathbf{V}}{2}\tilde{\mathbf{\Sigma}}_I\right], \tag{24}$$

where

$$I \in \operatorname*{argmax}_i \left\{m_i + \frac{1}{r}\left[\mathrm{Tr}\left[\frac{\partial^2 \mathbf{V}}{2}\tilde{\mathbf{\Sigma}}_i\right] + \mathrm{Tr}\left[\frac{\partial^2 \mathbf{U}_i}{2}\tilde{\mathbf{\Sigma}}_i\right] - \mathrm{Tr}\left[\frac{\partial^2 \mathbf{U}_i}{2}\tilde{\mathbf{\Sigma}}_{-i}\right]\right]\right\} \tag{25}$$

and

$$W = m_{-I} + \frac{1}{r}\left[\mathrm{Tr}\left[\frac{\boldsymbol{\partial}^2\mathbf{V}}{2}\tilde{\boldsymbol{\Sigma}}_{-I}\right] + \mathrm{Tr}\left[\frac{\boldsymbol{\partial}^2\mathbf{U}_{-I}}{2}\tilde{\boldsymbol{\Sigma}}_{-I}\right] - \mathrm{Tr}\left[\frac{\boldsymbol{\partial}^2\mathbf{V}}{2}\tilde{\boldsymbol{\Sigma}}_{I}\right] - \mathrm{Tr}\left[\frac{\boldsymbol{\partial}^2\mathbf{U}_{-I}}{2}\tilde{\boldsymbol{\Sigma}}_{I}\right]\right]. \tag{26}$$

Here $\boldsymbol{\partial}^2\mathbf{V}$ and $\boldsymbol{\partial}^2\mathbf{U}_i$ denote the matrices of second derivatives of V and U_i, $\mathbf{M}\tilde{\boldsymbol{\Sigma}}_i$ denotes the usual matrix product of the square matrices $\mathbf{M}$ and $\tilde{\boldsymbol{\Sigma}}_i$, and $\mathrm{Tr}[\mathbf{M}]$ denotes the trace of the square matrix $\mathbf{M}$.[22]

Equations (22)–(26) are identical to equations (10)–(14) above except that we have replaced terms of the form

$$\frac{\partial_i^2 V}{2}\Sigma_i, \frac{\partial_i^2 U_i}{2}\Sigma_i, \frac{\partial_{-i}^2 U_i}{2}\Sigma_{-i}$$

with terms of the form

$$\mathrm{Tr}\left[\frac{\boldsymbol{\partial}^2\mathbf{V}}{2}\tilde{\boldsymbol{\Sigma}}_i\right], \mathrm{Tr}\left[\frac{\boldsymbol{\partial}^2\mathbf{U}_i}{2}\tilde{\boldsymbol{\Sigma}}_i\right], \mathrm{Tr}\left[\frac{\boldsymbol{\partial}^2\mathbf{U}_i}{2}\tilde{\boldsymbol{\Sigma}}_{-i}\right].$$

D. The Pairwise-Team Problem

It will be helpful, for the results that follow, to have an explicit notation $\tilde{S}_i$ for the value function of pairwise-team problem *i*. The value function $\tilde{S}_i$ is the unique solution of the linear elliptic ordinary differential equation

$$\tilde{S}_i = m_i + \frac{1}{r}\frac{\tilde{S}_i''}{2}\alpha_i\Sigma_i.$$

It follows from Jensen's inequality that $\tilde{S}_i \geq m_i$ and that this inequality holds strictly on (0, 1) if $H_{iF} > L_{iE}$, that is, if fine art is more productive than economics when the worker has no aptitude for economics. This in turn implies that $\tilde{S}_i$ is convex and strictly convex if $H_{iF} > L_{iE}$.

E. Analysis of Equilibrium in the Dynamic Game

Theorem 2 continues to hold in the more general model.

THEOREM 5. Equilibrium exists and is efficient, and equilibrium payoffs are unique.[23]

[22] The trace of the square matrix

$$\mathbf{M} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}$$

is $m_{11} + m_{22}$.

[23] Once again it is possible to make substantial progress with forms of uniqueness beyond simple uniqueness of equilibrium payoffs. The problem is, however, more involved than in the basic model, and so we omit the details.

Proof. It follows from the work of Krylov (1980) that the Bellman equation

$$S = \max_i \left\{ m_i + \frac{1}{r} \mathrm{Tr}\left[\frac{\partial^2 \mathbf{S}}{2} \tilde{\mathbf{\Sigma}}_i \right] \right\}$$

for the value function S of the grand-team problem—which is a weakly nondegenerate, nonlinear elliptic partial differential equation—possesses a solution. See especially section 6.5 of that book. Moreover, it is easy to check that the Bellman equation

$$S_i = m_i + \frac{1}{r} \mathrm{Tr}\left[\frac{\partial^2 \mathbf{S}_i}{2} \tilde{\mathbf{\Sigma}}_i \right]$$

for the value function S_i of the pairwise-team problem i—which is a linear elliptic partial differential equation—possesses the solution $S_i(p_1, p_2) = \tilde{S}_i(p_i)$. These value functions can be used to construct equilibrium value functions and strategies as in the proof of lemma 4. Conversely, it can be checked that, given equilibrium value functions U_1, U_2, and V, the equilibrium value $S = U_1 + U_2 + V$ of the grand coalition solves the grand-team problem, and the equilibrium value $S_i = U_i + V$ of pairwise coalition i solves pairwise-team problem i. Hence U_1, U_2, and V are unique. Q.E.D.

F. Wage Dynamics

The grand-team problem of the general model of this section is not, unfortunately, a two-armed bandit problem. We cannot therefore give an explicit characterization of the equilibrium choice of employer I.[24] Formula (6) for the wage does, however, generalize.

THEOREM 6. The equilibrium wage can be written in the form

$$W = m_{-I} + \frac{1}{r} \mathrm{Tr}\left[\frac{\partial^2 \mathbf{S}_{-I}}{2} \tilde{\mathbf{\Sigma}}_{-I} \right] - \frac{1}{r} \mathrm{Tr}\left[\frac{\partial^2 \mathbf{S}_{-I}}{2} \tilde{\mathbf{\Sigma}}_I \right].$$

[24] The optimal policy I can in principle be found from the policy equation for the grand-team problem,

$$I \in \operatorname{argmax}_i \left\{ m_i + \frac{1}{r} \mathrm{Tr}\left[\frac{\partial^2 \mathbf{S}}{2} \tilde{\mathbf{\Sigma}}_i \right] \right\},$$

but we do not regard this as an *explicit* characterization. Although it is not possible to give an explicit characterization of I, it is possible to obtain a clear heuristic picture of I by interpolating between two special cases in which there is an explicit characterization of I. The first special case is the one in which $\alpha_1 = \alpha_2 = 1$. In this case, $I \in \operatorname{argmax}_i\{\psi_i\}$ as in the basic model. The only difference is that ψ_i will now be U-shaped if $H_{iF} > L_{iE}$. Hence regions 1, 2, and 3 may no longer be connected. The second special case is the one in which $\alpha_1 + \alpha_2 = 1$. In this case the pattern of learning is the same in both firms, the allocation of the worker is determined by purely static considerations, and $I \in \operatorname{argmax}_i\{m_i\}$. Like ψ_i, m_i will be U-shaped if $H_{iF} > L_{iE}$. Hence regions 1, 2, and 3 may once again not be connected.

In other words, as in the basic model, the equilibrium wage has three components: the worker's expected product m_{-I} in the alternative match, the value

$$\frac{1}{r}\operatorname{Tr}\left[\frac{\boldsymbol{\partial}^2\mathbf{S}_{-I}}{2}\tilde{\boldsymbol{\Sigma}}_{-I}\right]$$

to the alternative match of the accumulation of human capital specific to that match forgone in the course of the current match, and the value

$$\frac{1}{r}\operatorname{Tr}\left[\frac{\boldsymbol{\partial}^2\mathbf{S}_{-I}}{2}\tilde{\boldsymbol{\Sigma}}_{I}\right]$$

to the alternative match of the accumulation of human capital specific to that match obtained in the course of the current match.

Proof. This follows at once from (26) and the definition of S_i. Q.E.D.

The following two lemmas characterize the second and third components of the wage.

LEMMA 5. The second component of the wage

$$\frac{1}{r}\operatorname{Tr}\left[\frac{\boldsymbol{\partial}^2\mathbf{S}_{-i}}{2}\tilde{\boldsymbol{\Sigma}}_{-i}\right] = \tilde{S}_{-i} - m_{-i}.$$

Proof. We have

$$\begin{aligned}\frac{1}{r}\operatorname{Tr}\left[\frac{\boldsymbol{\partial}^2\mathbf{S}_{-i}}{2}\tilde{\boldsymbol{\Sigma}}_{-i}\right] &= \frac{1}{r}\frac{\partial^2_{-i}S_{-i}}{2}\alpha_{-i}\Sigma_{-i} + \frac{1}{r}\frac{\partial^2_i S_{-i}}{2}(1-\alpha_{-i})\Sigma_i \\ &= \frac{1}{r}\frac{\partial^2_{-i}S_{-i}}{2}\alpha_{-i}\Sigma_{-i} \\ &= \frac{1}{r}\frac{\tilde{S}''_{-i}}{2}\alpha_{-i}\Sigma_{-i} \\ &= \tilde{S}_{-i} - m_{-i}.\end{aligned}$$

Q.E.D.

LEMMA 6. The third component of the wage

$$\frac{1}{r}\operatorname{Tr}\left[\frac{\boldsymbol{\partial}^2\mathbf{S}_{-i}}{2}\tilde{\boldsymbol{\Sigma}}_{i}\right] = \frac{1-\alpha_i}{\alpha_{-i}}(\tilde{S}_{-i} - m_{-i}).$$

Proof. As in the proof of lemma 5, we have

$$\frac{1}{r}\operatorname{Tr}\left[\frac{\boldsymbol{\partial}^2\mathbf{S}_{-i}}{2}\tilde{\boldsymbol{\Sigma}}_{i}\right] = \frac{1}{r}\frac{\tilde{S}''_{-i}}{2}(1-\alpha_i)\Sigma_{-i}.$$

Moreover,

$$\frac{1}{r}\frac{\tilde{S}''_{-i}}{2}(1-\alpha_i)\Sigma_{-i} = \frac{1-\alpha_i}{\alpha_{-i}}\left(\frac{1}{r}\frac{\tilde{S}''_{-i}}{2}\alpha_{-i}\Sigma_{-i}\right) = \frac{1-\alpha_i}{\alpha_{-i}}(\tilde{S}_{-i} - m_{-i}).$$

Q.E.D.

A first application of lemmas 5 and 6 gives the following theorem.

THEOREM 7. Suppose that $H_{iF} > L_{iE}$ for $i \in \{1, 2\}$. Then the second component of the wage is strictly positive everywhere in the interior of the state space. Suppose, in addition, that $\alpha_i < 1$ for $i \in \{1, 2\}$. Then the third component of the wage is strictly positive everywhere in the interior of the state space.

In this theorem, the assumption that $H_{(-i)F} > L_{(-i)E}$ ensures that nontrivial task assignment takes place within firm $-i$, and the assumption that $\alpha_i < 1$ ensures that some information about the worker's aptitude for the two tasks that she can undertake in firm $-i$ accumulates even while she works for firm i.

Proof. If $H_{-iF} > L_{-iE}$, then it is easy to check that $\tilde{S}_{-i}(p_{-i}) \geq m_{-i}(p_{-i})$ for all $p_{-i} \in [0, 1]$, with strict inequality for all $p_{-i} \in (0, 1)$. The result is then immediate from lemmas 5 and 6. Q.E.D.

A second application of lemmas 5 and 6 gives the following lemma.

LEMMA 7. The equilibrium wage

$$W = \left(\frac{1-\alpha_I}{\alpha_{-I}} m_{-I} + \frac{\alpha_I + \alpha_{-I} - 1}{\alpha_{-I}} \tilde{S}_{-I}\right).$$

In other words, the equilibrium wage is an affine combination—that is, a linear combination in which the weights add up to one but are not required to be positive—of the worker's static outside option m_{-I} and her dynamic outside option $\tilde{S}_{-I}$.

Proof. This is immediate from lemmas 5 and 6. Q.E.D.

Two special cases of lemma 7 are of particular interest. First, if $\alpha_1 = \alpha_2 = 1$, then there are no spillovers. The worker is therefore paid her dynamic outside option $\tilde{S}_{-I}$. Second, if $\alpha_1 = \alpha_2 = 1/2$, then spillovers are such that the pattern of learning is the same in both firms. Dynamic considerations do not therefore enter the bargaining among the market participants, and the worker is paid her static outside option m_{-I}.

Lemma 7 also allows us to find conditions under which there are returns to tenure.

THEOREM 8. If $H_{iF} > L_{iE}$ and $\alpha_i < 1$ for $i \in \{1, 2\}$ and if $\alpha_1 + \alpha_2 > 1$, then there are positive returns to tenure everywhere in the interior of the state space.

In order to obtain a precise intuition for this result, it is helpful to write the equilibrium wage in the form

$$W = \tilde{S}_{-I} - \frac{1-\alpha_I}{\alpha_{-I}}(\tilde{S}_{-I} - m_{-I}).$$

That is, the equilibrium wage is the worker's dynamic outside option less a reduction that compensates for the information specific to the match with firm $-I$ that accumulates during the worker's tenure at firm I. Now, if $H_{iF} > L_{iE}$, then the worker's dynamic outside option $\tilde{S}_{-I}$ is strictly convex in p_{-I}. That is, information about the worker's aptitude in firm $-I$ has positive value within the context of the match with firm $-I$. On the other hand, $\tilde{S}_{-I}$ depends only on p_{-I}. Hence the worker's dynamic outside option will remain constant through tenure unless $1 - \alpha_I > 0$, that is, unless there is an informational spillover, in which case it will grow on average with time. The problem is that any informational spillover has a second effect: it results in a reduction $[(1 - \alpha_I)/\alpha_{-I}](\tilde{S}_{-I} - m_{-I})$ in the wage. This reduction is itself convex at all p_{-I} other than the break-even point b_{-I}, where it is concave. Hence it too grows on average over time unless $p_{-I} = b_{-I}$. If the rate $1 - \alpha_I$ of spillover is so large as to exceed the rate α_I of information accumulation in the context of a match with firm $-I$ itself, then the growth in the reduction exceeds the growth in the dynamic outside option, and the returns to tenure are therefore negative.

Proof. Note first that (i) the static outside option m_{-I} is a convex function of the belief p_{-I}, (ii) the dynamic outside option $\tilde{S}_{-I}$ of the worker is a strictly convex function of p_{-I} because $H_{iF} > L_{iE}$, and (iii) the equilibrium wage W is a strictly convex combination of the static and dynamic outside options because $\alpha_1 + \alpha_2 > 1$. Hence W is a strictly convex function of p_{-I}. But p_{-I} follows a nondegenerate martingale everywhere in the interior of the state space because $1 - \alpha_I > 0$. Hence W follows a strict submartingale.[25] That is, W is strictly increasing on average over time. Q.E.D.

VII. Concluding Remarks

The results of this paper show that it is possible to explain Topel's (1991) finding that there is a significant return to tenure over and above that attributable to total job market experience on the basis of a theory of firm-specific human capital. There are two main ingredients to such an explanation. First, the human capital specific to a match must be valuable in the context of that match. Second, the worker must be able to accumulate human capital specific to an alter-

[25] For an empirical test using longitudinal data as to whether wage residuals follow a martingale, see Farber and Gibbons (1994).

native match in the course of the current match. The first of these conditions seems uncontroversial. It must be stressed, however, that it is not fulfilled in the basic model. In that model the human capital specific to a match is valuable to the market, but it is not valuable within the context of the match itself. The second requires a little clarification. The human capital the worker accumulates is specific to the alternative match in the sense that it affects her productivity only in the alternative match, not in the sense that it can be accumulated only in the course of the alternative match.

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