

Job matching and the distribution of producer surplus

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Summary

We study wage determination in the Jovanovic model of matching, relaxing the standard assumption that wages continuously adjust to reflect on-the-job performance and studying aggregation of *ex-ante* heterogeneous career paths. We assume that workers have no bargaining power and consider an equilibrium where individual workers' age-earnings profiles are piecewise constant, reflecting their outside earning opportunities at each point in time. Turnover results from employers' firing decisions rather than from workers' quitting decisions, and the equilibrium delivers realistic cross-sectional and time-series implications. Employees receive only a portion rather than the whole of the *ex-ante* producer's surplus from established matches, and have individual incentives to lobby for increased job security. Inefficiently low aggregate turnover may result if such lobbying efforts are successful.

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1. Introduction

In the classic Jovanovic (1978) model of job matching and turnover, workers are paid their expected productivity (or actual production) at every instant in time. Their employer's profit flow is then identically equal to zero in expected terms, and turnover is driven by worker decisions to *quit* when current and expected future wages are too low relative to outside opportunities. As Jovanovic shows, the equilibrium timing of turnover is the same as long as wages follow stochastic processes which satisfy a zero-

expected-profit constraint. Hence, the model's turnover implications (and in particular the tendency of separation hazards to decline with tenure) are robust to assumptions regarding wage behaviour. The assumption that wage rates continuously adjust to reflect on-the-job performance also has observable implications: wage rates should (on average) increase with tenure on the current job. While empirical work focused on this implication finds that longer-lasting jobs pay higher wages along cross-sectional dimensions, the evidence is quite mixed on the longitudinal or time-series prediction that an individual worker's wages should increase with his own tenure. Many recent contributions conclude that, within a given employment relationship, wage dynamics are not systematically related to completed tenure (see, e.g. Abraham & Farber, 1987, and other references in Topel, 1991, who conversely argues that the prediction is borne out by longitudinal data when econometric problems are correctly addressed).

The theoretical and empirical robustness of Jovanovic's results to alternative assumptions regarding wage behaviour remains largely unexplored, and this motivates our work in this paper. Jovanovic (1984) admits that contracts which bestow all *ex-post* production rents to workers are not self-enforcing and probably unrealistic, and conjectures that the only perfect equilibrium would be an even less realistic one where none (as opposed to all) of producer's surplus accrues to workers. Workers would then be indifferent to all employment opportunities (including self employment, home production or leisure). If workers were not entitled to any portion of the *ex-ante* or *ex-post* surplus from productive matches, however, they would obviously be valuable to potential employers. In the absence of employers' collusion, a bidding process should lead to an equilibrium where workers receive compensation; depending on the nature of competition among employers, firms may or may not have zero value in equilibrium. These considerations lead us to characterize the market process by which surplus from established matches is split among employers and employees.

Our model features non-trivial aggregation of *ex-ante* heterogeneous stochastic career paths, rather than Jovanovic's simple juxtaposition of representative-agent problems. We suppose that a finite number n of employers forms wage-employment offers to every worker, and we take workers to have no bargaining power, so that their wages are determined by their outside option rather than by their performance on the job. We construct an equilibrium where workers choose the employment opportunity associated to the highest wage offer at every point in time. Wage paths are then constant through the worker's tenure with each employer. Since firms appropriate any excess of *ex-post* production over *ex-ante* expectations, labour market dynamics are firm-driven: employers can resolve an established employment relationship, and do so when

cumulative output gives evidence of a poor match, i.e. of low form-specific human capital. If all jobs are *ex-ante* equal for a given worker, as in Jovanovic (1979), then firms have zero value—but as a result of random fluctuations of production flows above and below a *constant* wage, rather than of instantaneous zero-profit constraints. If employers have different priors as to the worker's productive potential instead, as in Jovanovic (1984), then wage rates drop discretely upon match resolution in the equilibrium we propose, and employers receive a strictly positive portion of the *ex-ante* producer's surplus from the employment opportunities they control.

While in our framework the timing of turnover coincides qualitatively with that of Jovanovic's model, all job separations look like firing decisions: an individual worker's age-earnings profile is constant through tenure at a given job, and drops discretely upon match resolution if it changes at all. In cross-section, this wage structure associates higher wages to longer completed tenures, and in this respect the equilibrium we consider is not inconsistent with empirical evidence on wages and turnover. Our equilibrium also features non-trivial distribution of income across firms and workers, with interesting implications at the aggregate level. As both employers and employees attach positive value to established matches in general, the model can be used to analyse the role of job market structure, and of institutions such as job security provisions, in determining the shares of producer's surplus accruing to firms and to workers.

We outline the technological and informational structure of the labour market we consider in Section 2. The equilibrium wage and turnover process resulting from competition among employers for a given worker's informational human capital are characterized in Section 3 under the assumption of costless turnover, and in Section 4 allowing for positive turnover costs. Section 5 discusses the empirical implications of our equilibrium's wage structure, introduces job security provisions, and notes that individually rational incentives to obtain protection from dismissal may result in low-turnover equilibria. Section 6 concludes summarizing our results and outlining directions for further research.

2. Model outline

We consider a market where n symmetric firms, indexed by i , produce a single good with labour as the only factor of production. All market participants are risk neutral. Labour is supplied by a continuum of workers, and we normalize the size of the labour force to unity.

TECHNOLOGY

Firms competitively produce a single good using a stochastic

constant returns to scale production technology. Worker j is associated to an n -dimensional vector $(\mu_j^i; i = 1, \dots, n; j \in [0, 1])$, whose elements take values on the real line and index the productivity of his matches with each firm in the market. As in Jovanovic (1979, 1984), worker j produces

$$X_j^i(T_i) = \mu_j^i T_i + \sigma Z_j^i(T_i) \quad (1)$$

over a tenure of length T_i at firm i . The parameter μ_j^i denotes average productivity per unit time, but realized production is disturbed by the match-specific standard Wiener process $\{Z_j^i(T_i)\}$: the random variable $Z_j^i(T_i)$ is normally distributed with mean zero and variance T_i , and the increments $Z_j^i(T_i^1) - Z_j^i(T_i^0)$ are uncorrelated across non-overlapping time intervals and independent of production flows from other matches. The standard deviation per unit time of production's noise component σ is common knowledge. To simplify notation, we take it to be the same across jobs and across workers. This and other similar assumptions below might be relaxed without essential consequences.

Each firm can generate a perfectly elastic supply of jobs. The overall size of the market is determined by the measure of the labour force, and the size of any one of the n firms is determined by the measure of the workers it has been able to attract and finds profitable to retain at the wage rate they command in the labour market. With no capacity constraints on job creation, jobs are *not* "dam sites" in the sense of Akerlof (1981). Every employer is always ready to hire additional workers provided that their (expected) productivity over (expected) tenure weakly exceeds wage payments and other match-specific costs paid by the firm. This greatly simplifies the analysis. Since every type of job can be created at will, there is no competition among workers, and when deciding whether to hire or fire a given worker each firm can disregard the qualifications of other workers for the job. Hence, vacancies have zero value at the margin, and each individual worker's wage and career path depend only on his own (informational) human capital.

Every employment relationship entails a setup cost $K \geq 0$ in terms of output. This parameter represents firm-specific training, relocation costs, and other technological features. For notational simplicity, we let K be the same across all firms and for all jobs at each firm, and we take it to be paid regardless of whether the one under consideration is the worker's first job and of whether the worker is being hired by a new firm or is returning to a previous employer. The setup cost K is sunk and cannot be recouped upon dissolution of the match, and constitutes firm-specific (indeed, match-specific) investment as defined by Becker (1975).

INFORMATION STRUCTURE

We assume symmetric, incomplete information as to potential matches' productivity.† Workers and firms all observe the worker's characteristics and career performance, but this common information provides only noisy signals of match productivity.

When $T_i = 0$, workers and employers share the common view that μ_j^i are draws from a prior normal distribution.‡ We relax the Jovanovic (1979) assumption that all firms and all workers "look alike" before matching. The prior mean and precision parameters might both differ across firms and workers in general but, to fix ideas, we shall take precision to be constant at $\hat{h}(0)$, while the mean $\hat{\mu}_j^i(0)$ may differ across jobs i for a given worker j .§ As in Jovanovic (1979), observation of the cumulative production flow (1) provides additional match-quality information. This can be combined with the prior to obtain a posterior estimate of average productivity per unit time: after a tenure of length T_i ,

$$\hat{\mu}_j^i(T_i) \equiv \frac{\hat{h}(0)\hat{\mu}_j^i(0) + X_j^i(T_i)/\sigma^2}{\hat{h}(T_i)} \quad (2)$$

where

$$\hat{h}(T_i) \equiv \hat{h}(0) + \frac{T_i}{\sigma^2} \quad (3)$$

is the precision of the posterior distribution after observing the realization of $X_j^i(T_i)$. Applying Itô's lemma to (2) and (3), estimated productivity follows a process with stochastic differential

$$d\hat{\mu}_j^i(T_i) = (\hat{h}(T_i)\sigma)^{-1} dZ_j^i \quad (4)$$

as tenure T_i lengthens and production takes place. The process in (4) is driftless, reflecting unbiasedness of the productivity estimator in (2). Its diffusion coefficient decreased monotonically in T_i ,

† The symmetry assumption eliminates any need to use screening devices to induce the worker to reveal his private information.

‡ The assumption that both workers and employers share a common prior distribution is standard in the literature on incomplete information. The justification, known as "Harsanyi doctrine", relies on the fact that it is always possible to re-write a model in which agents have different priors as a model in which agents share a common prior and have different information sets. See Chapter 6, pp. 214–215 of Fudenberg and Tirole (1991) for an introduction to this doctrine.

§ In a companion paper (Bertola and Felli, 1993) we relax this assumption, and we study how the precision of prior information may be endogenously determined by the character of the schooling system which supplies potential employees to the job market.

reflecting the increasing quality of the information on which the estimator is based, and vanishes asymptotically reflecting consistency of the posterior productivity estimate. The market's equilibrium wage and turnover processes take these informational gains into account, and trade them off the opportunity of employing the worker in other, potentially more productive jobs.

LABOUR MARKET INTERACTIONS

We assume that firms behave competitively in the labour market, and that long term employment contracts are not enforceable. Rather, wages are continuously renegotiated on an individual basis, and workers supply their labour to the highest bidder among employers, deriving no disutility from work.† Firms may fire their current employees, who may symmetrically quit their current job at any point in time. Since firms can decrease wages at will, “quits” and “layoffs” are indistinguishable in realizations.

As in the classic Jovanovic (1979) model of job matching, the time profile of a worker's wage rates and the pattern of labour mobility depend on the evolution of the productivity estimate (2), reflecting on-the-job performance. Jovanovic (1979, 1984) assumes that competitive labour-contract offers satisfy a zero-value constraint for all jobs in existence, and shows that a possible market equilibrium would equalize wage rates and expected productivity per unit time. Jovanovic acknowledges that the one he considers is not the only possible competitive equilibrium, even among those that bestow to workers all rents from established matches. The assumption of competitive determination of *spot wage rates* might be awkward, in fact, when productivity is fully *firm-specific* and no employer other than the current one can offer a wage contract based on realized on-the-job performance. It is not clear what would force an individual (employer) to behave competitively when no competitors actually exist. In what follows we propose and characterize equilibria where the identity of employers formulating competitive wage offers is made explicit.

3. Equilibrium concept, and wage paths with costless turnover

Since other potential and actual employees' current and past performance are irrelevant to every firm's evaluation of productive potential, we can consider the career path of an individual worker of labour-market age t and omit the index j for now, with no loss of generality.

† This assumption eliminates any role for moral hazard.

We use the notation

$$\hat{\underline{\mu}}(t) \equiv \begin{pmatrix} \hat{\mu}_1(t) \\ \vdots \\ \hat{\mu}_n(t) \end{pmatrix}, \quad \underline{T}(t) \equiv \begin{pmatrix} T_1(t) \\ \vdots \\ T_n(t) \end{pmatrix} \quad (5)$$

for the $2 \times n$ state variables relevant to a worker's career path: a $1 \times n$ vector $\hat{\underline{\mu}}(t)$ of productivity estimates and a $1 \times n$ vector $\underline{T}(t)$ of cumulative tenures at the n firms which, by the Bayesian-updating rule (3), determine the volatility of productivity estimates and the probability distribution of their future realizations. These are sufficient statistics for the probability distribution of the worker's productivity in any one of the n firms at any time. As the worker is never unemployed, the elements of $\underline{T}(t)$ are non-negative and sum to t ; hence, the worker's age t is not a separate state variable. In particular, $\sum_{i=1}^n T_i(0) = 0$: a new entrant has no experience in any job, and productivity assessments about him are based on the prior distribution only.

Denote with

$$\underline{w}(t) \equiv \begin{pmatrix} w_1(t) \\ \vdots \\ w_n(t) \end{pmatrix} \quad (6)$$

the vector of wages offered by each firm $i \in \{1, \dots, n\}$ as a function of the worker's age t , which evolves like calendar time. The identity of the worker's employer is dynamically tracked by the vector

$$\underline{\chi}(t) \equiv \begin{pmatrix} \chi_1(t) \\ \vdots \\ \chi_n(t) \end{pmatrix} \quad (7)$$

where $\chi_i(t) = 1$ if the worker is employed by firm i , $\chi_i(t) = 0$ if he is not. A worker cannot be employed by more than one firm at each instant of time, so only one element of $\underline{\chi}$ is different from zero at a given time.

We shall rule out any binding commitments as to future payments or labour services, and seek an equilibrium of the labour market which satisfies the following definition:

DEFINITION 1: A *labour market equilibrium* is defined by $\underline{w}(t)$ and $\{\underline{\chi}(t)\}$ processes such that

- (E1) At every time t , $w_i(t)$ is chosen by firm i so as to maximize expected present discounted profits given: the state variables (5), other employers' wage offers, and the worker's decision rule.

(E2) The worker's current employer is the firm which extends the highest wage offer: $\chi_{i^*} = 1$ and $\chi_i = 0, \forall i \neq i^*$, where i^* indexes the highest element of $\underline{w}(t)$.

An equilibrium of this type is Markov,[†] in that each firm's wage offer and the allocation rule depend only on the payoff relevant state variables (5). Our equilibrium conditions may be interpreted in terms of a simple market allocation process: at every point in time, the worker is auctioned off to the highest bidder among the n potential employers. By property (E1), wage offers are Nash equilibria of a bidding game among potential employers, and by (E2) the worker is allocated to the employer who extends the highest offer.

The equilibrium property (E2) plays a central role in our model. Its role is symmetric to Jovanovic's (1979) instantaneous zero-expected-profit constraint, which makes workers the residual claimants to *ex-post* surpluses generated by every match. Our assumption (E2) has a similar role but opposite implications: workers have a completely passive role, hence employers are residual claimants to *ex-post* surplus. Accordingly, employers initiate all turnover in our equilibrium, while in Jovanovic's, workers take all mobility decisions. Neither assumption is beyond criticism, in that neither wage process is simultaneously self-enforcing from every agent's point of view. Jovanovic's instantaneous zero-expected-profit constraint is self-enforcing only if "firms" have no role in the market allocation process, i.e. if workers are self-employed. Our assumption (E2) would reflect self-enforcing contractual agreements between employers and employees only if workers myopically considered only current earnings opportunities when taking occupational decisions.

The case of costless turnover ($K=0$) lends itself easily to equilibrium analysis and provides useful guidance for more complex and realistic cases. Let $\hat{\mu}_{(i)}(t)$ denote the i -th order statistic of the vector $\hat{\mu}(t)$, so that (i) indexes the firm associated with the i -th highest element of $\hat{\mu}(t)$:

PROPOSITION 1: *If $K=0$, the wage offers*

$$w_{(i)}(t) = \hat{\mu}_{(i)}(t), \forall i \neq 1; \quad w_{(1)}(t) = \hat{\mu}_{(2)}(t) + \varepsilon \quad (8)$$

with $\varepsilon > 0$ but arbitrarily small, yield an equilibrium, satisfying Definition 1 with

$$i^* = (1), \quad (9)$$

[†] As defined in Chapter 13, pp. 529–536 of Fudenberg and Tirole (1991).

and wage

$$w(t) = w_{(1)}(t) = w_{i^*}(t) = \hat{\mu}_{(2)}(t) + \varepsilon.$$

PROOF: We need to show that the equilibrium described by (8) and (9) satisfies properties (E1) and (E2). Consider firm j , $j \neq (1)$. Given (E2), firm j 's *best response* to $w_{(1)}(t)$ is a wage $w_j(t) = \hat{\mu}_j(t)$. Since vacancies yield a zero cash flow, it is a *weakly dominated* strategy for firm j to offer a wage $w_j(t) > \hat{\mu}_j(t)$, which yields zero cash flow if $w_{(1)}(t) \geq w_j(t)$, or a negative cash flow if $w_j(t) > w_{(1)}(t) > \hat{\mu}_j(t)$. Consider now firm (1). Given (E2), its *best response* to firm (2)'s wage offer is $w_{(1)}(t) = \hat{\mu}_{(2)}(t) + \varepsilon$: firm (1)'s cash flow is zero if $w_{(1)}(t) \leq w_{(2)}(t)$, strictly positive and decreasing in $w_{(1)}(t)$ otherwise. We conclude that (E1) is satisfied. As to (E2), it is satisfied by the allocation rule (9) given wage offers (8).[†]

DISCUSSION

In the absence of turnover costs, the worker under consideration is employed by the firm which has the highest evaluation of his or her productivity, while the market wage reflects the second-highest evaluation (plus an arbitrarily small amount ε). The equilibrium characterization coincides with that of an English auction of the worker's performance for the following instant of time.[‡]

The market wage depends on the *second*-highest productivity estimate $\hat{\mu}_{(2)}(t)$, and two features of this simple equilibrium are worthy of notice. First, the equilibrium wage is determined by the worker's outside option, and is *constant* through tenure at a given job. The information provided by production flows is completely match-specific in the Jovanovic framework, hence only $\hat{\mu}_{i^*}$ (the current employer's productivity estimate) is updated over time. Second, the worker's wage reflects his expected productivity in an alternative (and less favourable) employment opportunity, rather than his actual or expected production in the current job. Hence, employers' profit flows are not zero in general. If different firms have different estimates for a given worker's productivity, competition in wage offers does not dissipate all of the employer's rents from established matches.

Turnover behaviour is completely characterized by the equilibrium wage offers in Proposition 1. The identity of the worker's

[†] This equilibrium is unique as far as the two highest wage offers are concerned, but there exists a multiplicity of equilibria where other firms' wage offers differ from those in (8). All these equilibria are equivalent on the equilibrium path. None of these equilibria is *trembling-hand* perfect except (8).

[‡] See McAfee and McMillan (1987) for the definition and equilibrium characterization of an English auction.

employer i^* in the equilibrium changes when the worker's performance leads the current employer to update $\hat{\mu}_{i^*}$ downwards to the point where $\hat{\mu}_{i^*}(t) = \hat{\mu}_{(2)}(t)$ at some t . The original employer is no longer the highest bidder for the worker's informational human capital and takes on the role of best outside option, while the worker chooses to be matched with the employer who was the second-highest bidder at time zero.† At this point the worker's estimated productivity is the same (up to ε) at more than one employer. By the infinite-variation property of the Brownian process driving realizations of production and estimated productivity, further turnover occurs immediately. The two estimated productivities remain arbitrarily close for a time interval of positive length during which infinitely frequent turnover takes place, and the wage is always given (up to ε) by the minimum of the two estimated productivities. Thus, the worker's equilibrium wage monotonically decreases by singular amounts until one of the two evaluations becomes discretely larger than the other; a tenure of finitely positive length follows.‡

Figure A illustrates a possible equilibrium wage and career path in the absence of turnover costs. Three firms' evaluations of the worker's informational human capital are plotted by lines marked with (respectively) circles, squares and triangles; the equilibrium wage path is plotted as a thick line and, by (8), always coincides with the second-highest among the three estimated productivities. At the beginning of his career, the worker is employed by the "circles" firm and is paid the valuation of the "squares" firm, but very soon the "circles" valuation falls enough to induce a switch to the "squares" job—where performance is bad, so that the worker is immediately re-employed by the "circles" firm and paid a reduced wage which reflects deterioration of the outside option offered by the "squares" firm. A similar episode shortly before $t=2$, and this time the relevant outside option upon returning to the "circles" firm is the wage offer of the "triangles" firm.

4. Equilibrium with costly turnover

We consider next an equilibrium for the labour market under consideration when job-switching costs are strictly positive

† Since the worker's true productivity becomes known asymptotically, turnover is sure to occur in finite time if $\mu_{(1)} < \hat{\mu}_{(2)}(0)$, but may never occur in other cases.

‡ A process is "singular" with respect to Lebesgue measure if its increments are infinitesimal and occur on a measure-zero, but dense set of time points. The locus of time points where a diffusion process takes on a pre-specified value (such as its running maximum) forms one such set. See Harrison (1985) and his references on p. 113 for an introduction to these concepts and further discussion.

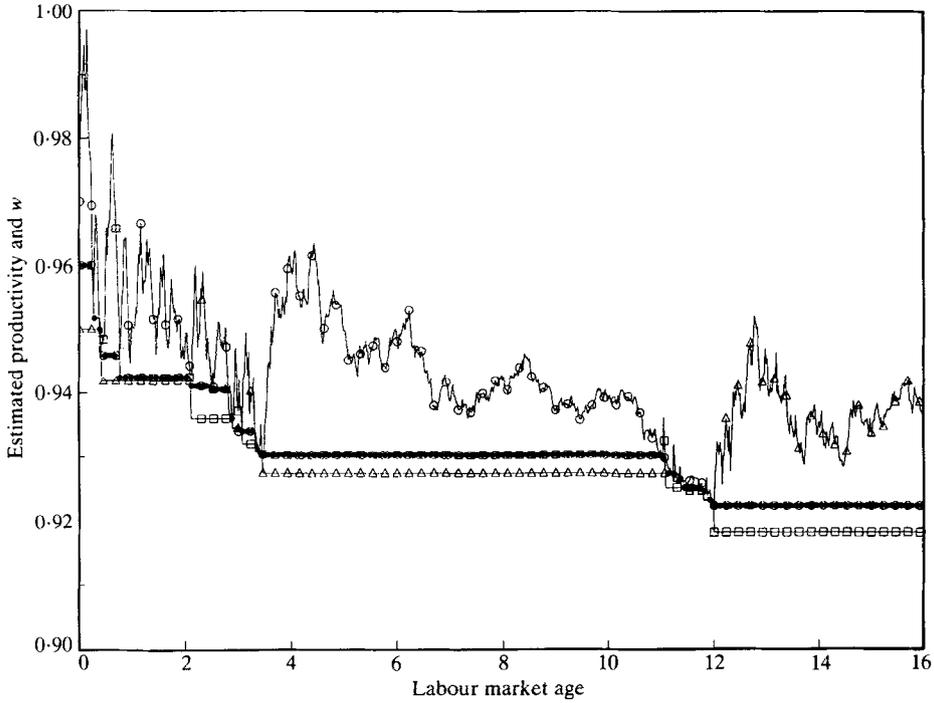


FIGURE A. Career path example (no turnover costs).

($K > 0$). We again take the worker to have no bargaining power, rule out binding commitments as to future wages and tenures, and seek an equilibrium which satisfies Definition 1. Accordingly, equilibrium wage and turnover paths must reflect the worker's instantaneous outside option, which is in turn determined by the (Markov) equilibrium strategies of potential employers.

By analogy to the equilibrium constructed above, we take wage rates to be independent of tenure at the current job. Since $\hat{\mu}_i(T_i)$ and T_i are sufficient statistics for the probability distribution of production flows and of termination time, the value to employer i of an established match with the worker under consideration can be written

$$\begin{aligned}
 V^*(\hat{\mu}_i(T_i), T_i; w) &= E_{T_i} \left[\int_{T_i}^{T_i^*} e^{-r(\tau - T_i)} (\mu_i d\tau + \sigma dZ_i(\tau) - w d\tau) \right] \\
 &= E_{T_i} \left[\int_{T_i}^{T_i^*} e^{-r(\tau - T_i)} (\mu_i - w) d\tau \right],
 \end{aligned}
 \tag{10}$$

for a constant (equilibrium) wage w , where T_i^* denotes the time at which employer i finds it optimal to terminate the match and leave the job vacant, r is the discount rate, and the second equality

follows from the fact that the stochastic integral in the first line of (10) has zero expectation because the (random) termination time T_i^* is a stopping time for the filtration generated by the Brownian motion process $\{Z(t)\}$.

We shall allow for a lump-sum *hiring cost*, H_i , paid by the employer to cover the cost of mobility. Employer i should be willing to hire the worker under consideration provided that

$$V^*(\hat{\mu}_i(0), 0; w) \geq H_i,$$

because in this case the established match has non-negative (and possibly positive) value to the firm.

It will be useful to implicitly define the maximum willingness to pay w_i^H as the (constant) wage rate that makes the employer indifferent between hiring a given worker or keeping a vacancy open:

$$V^*(\hat{\mu}_i(0), 0; w_i^H) \equiv H_i. \quad (11)$$

Employer i is willing to hire a given worker whenever the market wage w is less than or equal to w_i^H .

The employer's optimal firing policy is not as easily characterized as in the case of costless turnover. As finitely large turnover costs need to be paid at the beginning of each (re)-employment relationship, a job-worker match is interrupted for a finite-length period of time in equilibrium. The current employer must take this into account when evaluating the relative profitability of continued observation of the worker's firm-specific performance. The optimal termination policy trades off the option value of continued observation (and the possibility of an upward revision of the worker's estimated profitability) against possible current flow losses. The employer's optimal firing policy defines a *firing boundary* $\hat{\mu}^*(T_i; w)$ (see the Appendix) for which no exact analytical solution is available. Still, the state space of the firm's stopping problem may be redefined to yield a characterization similar to that discussed above. The maximum constant-through-tenure wage rate $w_i^f(T_i)$ that would make the employer indifferent to firing or retaining an incumbent worker (whose hiring cost is sunk) is implicitly defined by

$$V^*(\hat{\mu}_i(T_i), T_i; w_i^f(T_i)) \equiv 0. \quad (12)$$

For given optimal firing policy, $w_i^f(T_i)$ is uniquely defined as the wage rate that sets the firing boundary $\hat{\mu}_i^*(T_i; w)$ equal to the worker's estimated productivity $\hat{\mu}_i(T_i)$, to imply immediate termination. The dynamics of $\hat{\mu}^f(\cdot)$ are non-linearly reflected by (12) in the dynamics of $w_i^f(\cdot)$, which then measures in flow terms the

monetary value attached by an employer to the incumbent worker. It is optimal for an employer to retain the worker whenever $w_i^I > w$ (the flow value exceeds the wage rate), and to fire him when $w_i^I = w$. Quite obviously, if $H_i > 0$ then

$$w_i^H < w_i^I(0), \quad (13)$$

because with $w_i^H \geq w_i^I(0)$ the match would be terminated immediately and the integral defining V^* would be identically zero rather than H_i as required by the definition of w_i^H .

Accordingly, we redefine (i) as the index of the firm associated to the i -th largest element of the vector \underline{w}^H defined by (11) for $i = 1, \dots, n$, and propose an equilibrium based on the willingness-to-pay construct (11) in the following

PROPOSITION 2: *When $K > 0$, let every employer pay the mobility cost up front, so that $H_i = K$ for all i , and consider a worker's first spell of tenure with a given employer. For any $T_i < T_i^*$ and corresponding job market age t the constant-through-tenure wage offers*

$$w_{(i)}(t) = w_{(i)}^H, \quad \forall i \neq 1; \quad w_{(1)}(t) = w_{(2)}^H + \varepsilon \quad (14)$$

(with $\varepsilon > 0$ but arbitrarily small) sustain an equilibrium satisfying Definition 1 with

$$i^* = (1), \quad (15)$$

and wage

$$w(t) = w_{(1)}(t) = w_{i^*}(t) = w_{(2)}^H \varepsilon.$$

When the initial match is resolved at T_i^* , the worker's career path and the equilibrium wage are determined by similar expressions: the wage rates $w_{(i)}^H$ are computed on the basis of (11) for every firm i except the former employer i^* , whose maximum willingness to pay is implicitly defined by an expression similar to (11) but computed at tenure $T_i^* > 0$.

PROOF: Adapting the arguments in the Appendix, $\partial V^*(\hat{\mu}_i(0), 0; w) / \partial w < 0$. Hence, each potential employer wants to keep his bid for the worker's services as low as possible. As willingness-to-pay w_i^H summarizes the employer's dynamic evaluation of the worker's services, properties (E1) and (E2) can be verified as in the proof of Proposition 1 if $\hat{\mu}_{(i)}(t)$ is replaced by $w_{(i)}^H$; the wage offers are Markov, because w_i^H is fully determined by the vector of state variables (5). As to the lump-sum payments $H_i = K$, given the wage offers it is a *weakly dominated strategy* for each of the n firms to

offer $H_i > K$; if accepted, such an offer would strictly decrease the value of the match at inception. By the allocation rule (E2), the worker will not accept an employment offer if it entails up-front relocations expenses; given $H_j = K$, it is then weakly dominated for firm $i \neq j$ to offer $H_i < K$: this leaves payoffs unchanged if the worker was not going to be hired, and yields a strictly lower payoff if, by (E2), it induces the worker to decline an employment offer which had positive value from employer i 's point of view.

DISCUSSION

In equilibrium, the current employer pays the wage rate which would yield zero value for one or more of the other possible matches, and a positive value for none. Other potential employers stand ready to offer employment at a wage which makes them indifferent to filling the potential vacancy, paying the worker's switching cost, or keeping it empty. The worker is never called upon to pay match-specific mobility costs since, as in Becker (1975), his lack of bargaining power would *ex-post* make it impossible for him to appropriate any portion of *ex-post* match-specific returns.

At T_i^* , when the worker relocates, the wage is set at the level that would make the next bidder's value zero. In general, different employers have different priors as to the worker's productivity, and the worker is initially allocated to the employer with highest willingness-to-pay. Hence, the new wage rate does not coincide with the wage previously received by the worker, and the age-earnings profile of the worker is piecewise constant with infrequent, discrete downward jumps in place of the continuous and monotonically decreasing portions of the wage path illustrated in Figure A. Figure B illustrates the model's dynamics, again displaying three firms' maximum willingness to pay for the same incumbent worker (only one of which is being updated at any given time) along with the equilibrium wage. While in Figure A the wage path coincided with the second-highest productivity estimate $\hat{\mu}_{(2)}$ at every point in time, by equation (13) the wage lies *below* the second highest maximum willingness to pay for an incumbent worker $w_{(2)}^I$ supporting the forward-looking outside wage offer in Figure B, where $K > 0$.

The discontinuities in the wage path occur at those points in the $\{\underline{w}^I, \underline{T}_i\}$ state space which correspond to points where match termination is optimal for the current employer. Optimal termination points depend on the nature of the wage process in turn, and the equilibrium of Proposition 2 identifies a fixed point of this mapping. As firing policies, wages, and the allocation rule only depend on the state variables, the equilibrium's fixed point is Markov.

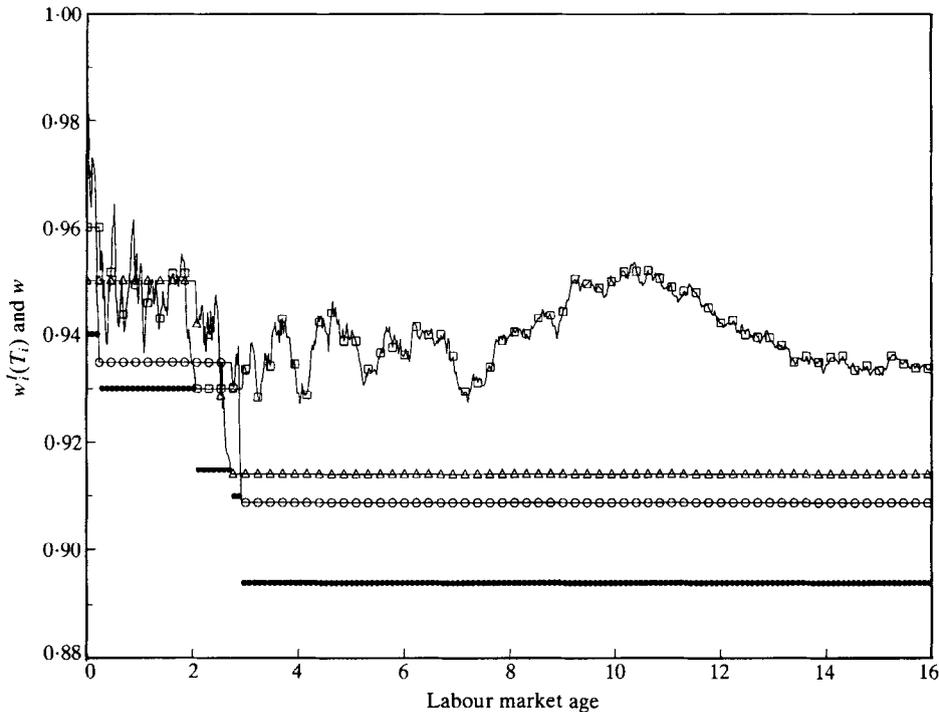


FIGURE B. Career path examples (costly turnover).

The equilibrium described in Proposition 2 has some appealing features: it allows each employment relationship to be treated in isolation from past and especially future relationships, and delivers a number of realistic theoretical and empirical implications which we discuss in the next section.

5. Implications and aggregation

In the equilibrium of Proposition 2, the market allocates workers to the highest bidders among potential employers, up-front payments by firms make mobility *costless* for workers, and individual wage rates are constant through tenure. As workers have no bargaining power, an established match has positive value from the point of view of the employer if a worker's expected productivity is *a priori* different across potential employers: the current one need only pay an arbitrarily small ε above the highest competing offer to obtain the worker's labour services and informational human capital.

This section first discusses the empirical implications of our equilibrium's age-earnings profiles, then notes that, since our equilibrium features non-trivial sharing of established matches'

producer surplus, interesting insights may be gained into such labour market institutions as individual job security.

For both purposes, it will be useful to be explicit as to the aggregative structure of the labour market being considered. We shall take the labour market of our n firms to be populated by a continuum of workers of unitary total size, and assume that constant inflows and outflows of workers assign negligible probability to the event that any individual productivity μ_i^j be learned with certainty through perpetual employment. Each worker “dies” (or leaves the market) with constant probability intensity δ , independently of all other stochastic elements of the model. An inflow of δ new market participants per unit time keeps the labour force constant at unity (similar assumptions are in Jovanovic, 1978). As workers form a continuum, the realized distribution of $\hat{\mu}_i^j(0)$ may be taken to be stable over time.† Further, the stable distribution may be taken to be symmetric across firms and workers, so that every firm acts as the initial employer and as the relevant outside option for the same, constant proportion of job-market entrants. Strong independence assumptions also let the joint distribution of productivity and residual life be decomposed in two multiplicative components. Firms’ and workers’ forward-looking considerations then feature expectations, over the probability distribution of productivity only, of cash flows discounted at rate $r + \delta$ rather than at rate r .

Given the previous sections’ characterization of labour markets dynamics, new entrants seek an occupation at a continuum of production sites, or “jobs,” located at the n firms introduced in Section 2 above. Job losers are similarly allocated to the highest-paying job among those offered by firms other than the most recent employer. Badly mismatched workers are likely to move more than once before “dying”, possibly returning to former employers (at lower wages) after a spell of employment in other firms. Over time, our model’s agents learn about the unknown vectors $(\mu_j^i; i = 1, \dots, n; j \in [0, 1])$ of true productivities. The learning process resembles a continuous-time multi-armed-bandit selection strategy.‡

In a steady state of this type, probability distributions coincide with cross-sectional distributions and expectations coincide with

† The notion that all idiosyncratic uncertainty washes out in a continuum of random variables is intuitively appealing and can be made formally precise along the lines of Judd (1985); laws of large numbers apply to realized or *empirical* distributions, as well as to sample moments, by the Glivenko-Cantelli theorem (see e.g. Billingsley, 1986).

‡ Miller (1984) and McCall (1991) use a multi-armed-bandit framework to study labour mobility decisions across occupations which differ in terms of the location and dispersion of prior *wage* distributions. Our framework is quite similar in many respects but, to derive wages from underlying *productivity* distributions, we need to specify whether firms which are in a position to offer jobs in a given occupation are independently managed or not.

sample means. Hence, we can discuss the properties of the aggregate equilibrium in terms of realized (and in principle observable) productivity, tenure length, and income distribution as well as in *ex-ante*, probabilistic terms.

MEASURED "RETURNS" TO TENURE

In the Jovanovic (1979) equilibrium, a worker's wage reflects his performance on the job. Mismatched workers experience decreasing wages and may eventually quit, bearing turnover costs, to sample another employment opportunity which has the same *ex-ante* characteristics as the one that has *ex-post* been learned to be of inferior quality. As low-wage workers quit and high-wage workers stay, the realized wage is on average increasing with tenure for a cohort of workers. In a steady-state like the one we consider, where all cohorts of market entrants are alike, the same implication would hold for a random cross-sectional sample of realized tenures to date and wage rates. Wages are indeed increasing with tenure in most occupations after controlling for age, education, and other observable characteristics (see, e.g. Becker, 1975). The original Jovanovic contribution de-emphasizes wage dynamics, and its results on the equilibrium timing of turnover are robust to such assumptions, as shown in his Proposition 2 and we show in some more generality below. Yet, the empirical evidence might be taken to provide empirical support not only for the relevance of match-specific informational human capital, but for the specific assumptions as to wage behaviour as well.

In the equilibrium of Propositions 1 and 2, however, the time profile of wages for an individual worker is quite different from that assumed by Jovanovic. Still, the cross-sectional implications of the wage structure we propose are consistent with positive "returns to tenure" if employment opportunities are *ex-ante* distinct from each other (so that, as in Figures A and B, the wage decreases when the worker changes jobs). To see this, consider two observationally equivalent workers entering the job market at the same calendar time, and suppose that only one of them has experienced job changes at the time when the cohort is observed: his wage will have dropped below the (common) initial level on the one hand, and his realized tenure will be shorter than that of his counterpart on the other.

When data on an individual worker's age-earnings profile are considered, conversely, our equilibrium's implications are quite different and arguably more realistic than those of Jovanovic's one. Within each employment relationship, the wage path is *constant* and hence displays no systematic tendency to increase with tenure: this may well rationalize the rather elusive empirical

evidence on longitudinal “returns to tenure” (see, e.g. Abraham & Farber, 1987; Topel, 1991).

As to the effect of turnover on wage behaviour, wages typically undergo a step *increase* upon match termination in Jovanovic’s quit-driven equilibrium: since workers finance turnover costs, for a quit to be optimal wages in the new job must be expected to be higher than in the old one over the relevant future, and by the independent-increments property of the process in (4) this usually requires that the wage earned in the new job be initially higher than the old one.† In the equilibrium of Proposition 2, conversely, wages *fall discretely* when turnover is triggered by disappointing performance on the current job and the worker moves to the employment opportunity which was *a priori* second-best. The empirical counterpart of our model’s total separations is *firings*, in the sense that employers always initiate termination of employment relationships. While quits and layoffs are formally indistinguishable in our model, every separation is triggered by the current employer—who either reduces the wage offer below the worker’s outside option, or fires him outright: the worker would rather continue the employment relationship at a wage rate reflecting his *outside* opportunities. Empirical evidence indicates that workers who are singled-out for termination do earn lower wages upon re-employment (see, e.g. Bartel & Borjas, 1981; Gibbons & Katz, 1991; Flinn, 1991). While this could be explained by asymmetric information across employers as in Gibbons and Katz (1991), or by moral-hazard as in Flinn (1991), our equilibrium offers an interpretation of this empirical fact when wage dynamics and turnover are driven by accumulation of *symmetric*, match-specific information.‡

AGGREGATE EFFICIENCY

Given our aggregative structure, we may briefly consider the efficiency properties of Proposition 2’s equilibrium. Recall that workers, for a given mobility cost $K = H_i$, are allocated to the employer extending the highest wage offer: this is based on willingness-to-pay as defined in equation (11), and the allocation will be efficient if the firm’s value function correctly internalizes aggregate objectives.

† Under the Jovanovic assumptions, wages can decrease upon job change only if turnover costs are small and the new employment opportunity offers a highly volatile wage process.

‡ In reality, of course, total separations include collective *layoffs*, triggered by product market shocks, and also true *quits*, motivated by non-stationary outside opportunities for workers. Our model, like Jovanovic’s, abstracts from all such features.

Our simple equilibrium does not deliver the result in general. To see this, consider that the employer's maximization problem in (10) may be written:

$$\max_{T_i^*} E_{T_i} \left\{ \int_{T_i}^{T_i^*} \mu_i e^{-(r+\delta)(\tau-T_i)} d\tau - \frac{1}{r+\delta} \left[1 - e^{-(r+\delta)(T_i^*-T_i)} \right] w_i \right\},$$

or equivalently (since w_i is taken as given and determined by the outside offer)

$$\max_{T_i^*} E_{T_i} \left\{ \int_{T_i}^{T_i^*} \mu_i e^{-(r+\delta)(\tau-T_i)} d\tau + \frac{w_i}{r+\delta} e^{-(r+\delta)(T_i^*-T_i)} \right\}. \quad (16)$$

Consider then the optimization problem facing a hypothetical social planner, who would be concerned with total discounted expected surplus (net of turnover costs): if $W(\underline{\hat{\mu}}(t), \underline{T}(t), i)$ is defined as the social value function at time t when the worker is allocated to firm i , then the following Bellman recursion holds true:

$$\begin{aligned} W(\underline{\hat{\mu}}(T_i), \underline{T}(T_i), i^*) \equiv \\ \max_{T_{i^*}^*} E_{T_{i^*}^*} \left\{ \int_{T_{i^*}^*}^{T_{i^*}^*} \mu_{i^*} e^{-(r+\delta)(\tau-T_{i^*}^*)} d\tau + \right. \\ \left. \left[W(\underline{\hat{\mu}}(T_{i^*}^*), \underline{T}(T_{i^*}^*), i^{**}) - K \right] e^{-(r+\delta)(T_{i^*}^*-T_{i^*}^*)} \right\}. \end{aligned} \quad (17)$$

The optimization strategy from time T_i^* onwards (and the resulting value function) are of course taken as given by the social planner when choosing T_i^* . Accordingly, the problem solved by our equilibrium's employer and the social optimum coincide if the same maximand appears on the right-hand sides of (16) and (17). This is the case if

$$W(\underline{\hat{\mu}}(T_{i^*}^*), \underline{T}(T_{i^*}^*), i^{**}) = \frac{w_{i^*}}{r+\delta} + K, \quad (18)$$

i.e. if the market wage is the annuity value of the social surplus that the worker would optimally generate along a career path starting with the employer which is providing the outside offer in market equilibrium.

Consider then how the market wage is determined in our equilibrium. As the outside offer sets to zero the optimized match value for the employer extending it, we have:

$$w_{i^*} E_0 \left[\int_0^{T_{i^*}^*} e^{-(r+\delta)\tau} d\tau \right] + K = E_0 \left[\int_0^{T_{i^*}^*} \mu_{i^*} e^{-(r+\delta)\tau} d\tau \right]. \quad (19)$$

It is easy to see that the capitalized value of the equilibrium wage, plus the lump sum turnover cost, coincides with social surplus if $T_{i^*} = \infty$ with probability one. More interestingly, the same obtains if further turnover would not have effects on the worker's estimated productivity, i.e. if there exists a fringe of infinitely many firms with the same prior information on the worker under consideration. To see this, consider in the latter case the Bellman recursion (17) defining $W(\hat{\mu}_\rightarrow(T_{i^*}^*), \underline{T}_\rightarrow(T_{i^*}^*), i^{**})$, and let $\{\hat{T}^h\}$ denote the sequence of optimal turnover times as the worker is employed by each of the fringe firms in turn, $h=1, \dots, +\infty$. If turnover times \hat{T}^h solving the sequence of employers' maximization problems (16) coincide with those solving the social planner's problem (17), recursive substitution yields

$$W(\hat{\mu}_\rightarrow(T_{i^*}^*), \underline{T}_\rightarrow(T_{i^*}^*), h=1) = E_{T_{i^*}^*} \left[\int_{T_{i^*}^*}^{+\infty} \mu_1 e^{-(r+\delta)(\tau - T_{i^*}^*)} d\tau - K \sum_{h=1}^{+\infty} e^{-(r+\delta)(\hat{T}^h - T_{i^*}^*)} \right], \quad (20)$$

where, by assumption, $\mu_1 = \mu_h$ as h indexes firms in the fringe. Consider next the discounted sum at $T_{i^*}^*$ of the two sides of outside-option conditions (19) for the infinite firms in the fringe:

$$\frac{w_{i^*}}{r+\delta} + K = E_{T_{i^*}^*} \left[\int_{T_{i^*}^*}^{+\infty} \mu_1 e^{-(r+\delta)(\tau - T_{i^*}^*)} d\tau - K \sum_{h=1}^{+\infty} e^{-(r+\delta)(\hat{T}^h - T_{i^*}^*)} \right], \quad (21)$$

where we again use $\mu_1 = \mu_h$ for every h indexing firms in the fringe. The assertion above follows, recognizing that (20) and (21), with the same right-hand side, yield (18).

In particular, the capitalized value of equilibrium wages, plus the turnover cost, coincides with social surplus if, as in Jovanovic (1979), *all* firms and all workers "look alike" before matches are established. In this case, Bertrand competition in wage offers bids firms' values down to zero, workers appropriate all of the social value of their labour, and our framework of analysis coincides with that of Jovanovic (1979) in an *ex-ante* sense. Hence, our equilibrium provides an alternative interpretation of Jovanovic's equilibrium: regardless of wage dynamics, *ex-post* turnover behaviour is socially optimal if social returns to labour mobility are correctly internalized by decentralized turnover decisions, whether taken by workers (in Jovanovic) or by employers (here).

In general, however, a given worker's wage adds up to only the *second*-highest career path's social production in expected present discounted terms. The market wage is based on the stochastic characteristics of the firm extending the outside offer *only*, neglect-

ing the valuation of potential employers further down the line: not surprisingly, the essentially myopic behaviour of workers in our equilibrium makes it impossible in general to guarantee that the market outcome is socially efficient. Interestingly, however, a realistic relaxation of representative-agent assumptions implies that the social planner's objectives no longer coincide with a typical worker's. We turn to this in the next subsection.

JOB SECURITY

Workers would of course like to appropriate as much as possible of the expected production flow they are expected to generate. In the equilibrium we analyse, workers appropriate more of expected producer's surplus if the parameters of the market allocation process are such as to excite more intense competition for their informational human capital. Quite intuitively, workers would like *ex-ante* willingness to pay to be as similar as possible across potential employers.

In this paper we focus on incentives to obtain protection from performance-related dismissal. Workers have incentives to delay or eliminate the downward wage jump that occurs upon dismissal, and their desire to do so may rationalize imposition of "job security" provisions on their current employer. Legal restraints on employers' freedom to dismiss are indeed quite common in reality. Individual dismissal legislation protects workers from being "unfairly" fired in many European countries. While gross misconduct or lack of qualification on the part of the employee would in principle allow for summary dismissal without compensation, it requires expensive labour court procedures if the fired worker appeals (Bentolila & Bertola, 1990); in practice, out-of-court settlements and severance bonuses are common in such institutional settings.

We model such features imposing that a cost F be paid by the employer upon match termination to a third party.† Still taking wages to be constant through tenure, employer i 's objective function (10) is then replaced by

$$V^*(\hat{\mu}(T_i), T_i; w) = E_{T_i} \left[\int_{T_i}^{T^*} e^{-r(\tau - T_i)} (\mu - w) d\tau \right] - E_{T_i} \left[e^{-r(T^* - T_i)} \right] F, \quad (22)$$

† Redundancy payments may at least in part be paid to individual workers instead. Such payments are not self-enforcing, of course, and would need to be state-mandated or specified by enforceable state-contingent contracts. Incentives to obtain them would be similar to those characterized below, but in symmetric equilibrium wage rates and/or hiring payments would adjust to fully offset them in present discounted terms as in Lazear's (1990) static model.

where T^* denotes the time at which the employer finds it optimal to terminate the match and pay F (see Appendix). A constant-through-tenure willingness to pay w_i^H can still be defined from (11) and the new definition (22) of V^* ; the willingness to pay for an incumbent worker $w_i^I(T_i)$ may then be implicitly defined from

$$V^*(\hat{\mu}(T_i), T_i; w_i^I(T_i)) \equiv -F \quad (23)$$

as the maximum constant-through-tenure wage rate that would make the employer indifferent between firing (at cost F), or retaining an incumbent worker (whose hiring cost is sunk).

We proceed to consider equilibria similar to that of Proposition 2 for positive F : the worker is employed by the firm whose willingness to pay w_i^H is initially the highest, and earns a wage reflecting the highest among other (outside) wage offers. The optimal firing policy is isomorphic to that discussed above, and the model's dynamics are similar to those illustrated in Figure B.

The emergence of job security provisions may be formalized in terms of a politico-economic game between employer and employees, taking place when workers know their *ex-ante* employment opportunities but are ignorant of their *ex-post* performance and of their future turnover path. The following characterizes the incentives to obtain job security from an individual worker's point of view:

PROPOSITION 3: *In ex-ante terms each worker strictly prefers the equilibrium outcome of Proposition 2 when the firing cost of his current employer is increased, but not enough to modify the identity of his current employer.*

PROOF: We show in the Appendix that a larger firing cost F increases tenure length for a given wage rate, and decreases the match's value to the employer and his willingness to pay. In the equilibrium of Proposition 2, however, the current employer's value function has no role as long as his willingness to pay remains above that of the second-highest bidder. Hence, workers have incentives to lobby for job-security provisions to be imposed on their employer: a higher F imposed on firm j^* increases tenure at the initial high-wage job, has no effect on the wage rate, and has heavily discounted effects on future career prospects.

DISCUSSION AND POSSIBLE EQUILIBRIA

Employers, who are the residual claimants to the *ex-post* benefits of more efficient matching, should of course resist imposition of firing

costs on their firms (but would not mind imposing firing costs on *other* firms, since these reduce the employee's outside options and wage rates). We do not explicitly model the bargaining structure of the game which determines job security provision, nor the role of "firms" in it. Clearly, however, workers initially employed by the firm which extends the outside offer have incentives similar to the ones described in Proposition 3. If efforts to obtain protection from dismissals are uncoordinated among workers, lower equilibrium wages will be needed to keep the value of outside wage offers at zero. If all workers succeed in imposing legal dismissal costs on their employer, all outside offers and all equilibrium wages will be uniformly lowered in a symmetric Nash equilibrium.

We show in the Appendix that, from the employer's point of view, the lower wage rate that preserves the initial value of the match at zero in the presence of higher F is not enough to induce firing at the same level of estimated productivity once the firing boundary is approached. Hence, turnover is unambiguously less frequent in an equilibrium with higher firing costs (stricter job security). Since the evolution over time of $\{X(t)\}$ is unaffected by w and F , longer average duration of low-performance matches implies that, on average, less is produced in the equilibrium under consideration when F is larger. These aggregate losses, however, do not necessarily make high job security unattractive to individual workers or to categories of workers. Currently employed workers or "insiders" should try and obtain high job security if they have the political power to do so, and uncoordinated lobbying for increased job security may quite possibly result in very inefficient equilibria with high all-around job security. It is important to recognize, however, that efficiency as such is not the objective of workers. Employees as a category may prefer low-turnover equilibria where they get a large share of a small aggregate production to high-turnover, high-aggregate production equilibria where their share is small.

6. Concluding comments

We have considered an equilibrium for the Jovanovic matching model where wage rates are fully determined by the employers' evaluation of the worker's (informational) human capital. Inasmuch as on-the-job performance provides only match-specific information, individual workers' earning profiles are non-increasing and piecewise constant over a worker's lifecycle, with downward jumps corresponding to firing decisions based on poor performance on the job. In an equilibrium of this type, short completed tenures are associated to previous job losses for a worker of given job-market age. The model delivers the positive relationship between

wages and tenure observed in cross-sectional data if *ex-ante* productivity evaluations differ across firms, so that job loss entails a wage cut in our equilibrium. When an individual worker is observed over time, wage cuts associated to job changes are also realistic in light of evidence that dismissed workers earn lower wages upon re-employment.

The model allows for a non-trivial distribution of income across employers and employees. Competition in wage offers among firms which control imperfectly substitutable employment opportunities for a given worker need not dissipate all of the employer's rents from an established match: the successful bidder for an employee's human capital need only pay a wage reflecting the *second highest* estimate of the worker's productive potential. In this paper, we have briefly explored the implications of this feature, noting that workers have incentives to obtain institutional protection from dismissal and that such efforts, if successful, will result in inefficiently high turnover costs and socially suboptimal turnover.

Our approach to wage determination in matching models suggests many directions for further research. Two of our assumptions are responsible for the character of wage profiles in our labour-market specification: first, that the information provided by on-the-job performance is purely firm specific and, second, that firms have full bargaining power in the labour market. While these assumptions afforded substantial simplifications in our formal analysis above, it would of course be desirable to relax them in future work, especially in light of evidence provided by Farber and Gibbons (1991) as to the relevance of informational mechanisms in wage dynamics. If performance at a specific job were allowed to provide at least some general productivity information, a worker's outside option would evolve with tenure in ways that are related to his performance, and our model could rationalize the returns to tenure measured by Topel (1991) as well as their elusive nature in other empirical work. Similarly, if the worker had some power in the wage bargain then wage rates would (partly) reflect on-the-job performance. In the limit where workers have full bargaining power and can appropriate all of the surplus from an employment relationship, the equilibrium we propose would be indistinguishable from Jovanovic's. If information is at least partly firm-specific and firms are granted at least some bargaining power, however, employers' decisions to fire low-performance employees will still be associated to general human capital losses, and to decreasing segments or step jumps in workers' age earning profiles.

The equilibrium concept we introduce in Definition 1 relies on a market mechanism that at each instant of time allocates a worker to the firm which extends the highest wage offer. The resulting behaviour of the worker cannot be rationalized as the optimal strategy choice of a fully rational, forward looking individual. In

general, if workers are active players in the multi-armed-bandit allocation process they might find it optimal to work for lower current wages, with an eye to improve future outside options. The interaction of workers' and employers' optimal experimentation policies defines an extremely intricate fixed-point problem in stochastic processes, which we found impossible to solve. In future research, it might be possible to devise appropriate simplifying assumptions and obtain a tractable self-enforcing equilibrium wage and allocation process.

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Appendix: firing costs

To ease notation we shall focus on one worker/firm match and omit the i and j indexes in this appendix. Further, we shall focus on the worker's first match so that t indexes both labour market age and tenure with the current employer.

When turnover is costly, the firm's optimal firing policy is symmetric to the worker's quitting policy in Jovanovic (1979). Under the usual differentiability and boundedness conditions (see Jovanovic: Thm. 4, p.984) the match's value satisfies the differential equation

$$rV^*(\hat{\mu}, t, w) = \hat{\mu} - w + \frac{\delta}{\delta t} V^*(\hat{\mu}, t, w) + \frac{1}{2} (\hat{h}(t)\sigma)^{-2} \frac{\delta^2}{\delta \hat{\mu}^2} V^*(\hat{\mu}, t, w)$$

in the *continuation region* where the employer takes no action. If an optimal policy exists and is unique, the continuation region takes the form $\{\hat{\mu}(t) \geq \hat{\mu}^*(t)\}$ in the $(\hat{\mu}, t)$ state space of the firm's problem, and the conditions

$$V^*(\hat{\mu}^*(t), t, w) = -F \tag{A.1}$$

$$\frac{\delta}{\delta t} V^*(\hat{\mu}^*(t), t, w) = 0 \tag{A.2}$$

must be satisfied on the boundary $\hat{\mu}^*(t)$ of that region, where *firing* occurs. Note that the characterization (A.1) and (A.2) of an employer's optimal firing policy applies both to the case $F=0$ and the case $F>0$.

When $F > 0$, recognizing explicitly the dependence of the employer's value function and of the optimal policy on the parameter F , and denoting with $f(T^*; F)$ the density induced on $T^* \in [t, \infty]$ by the optimal firing policy identified by (A.1) and (A.2) for given firing cost F , we have from (17)

$$V^*(\hat{\mu}(t), t; F) = \int_t^\infty \left[\int_t^{T^*} e^{-r(\tau-t)} (\mu - w) d\tau - F e^{-r(T^*-t)} \right] f(T^*; F) dT^* \quad (\text{A.3})$$

By the *envelope* property in Jovanovic (1979; Thm. 6, p.986),

$$\frac{d}{dF} V^*(\hat{\mu}(t), t; F) = - \int_t^\infty e^{-r(T^*-t)} f(T^*; F) dT^*. \quad (\text{A.4})$$

The density $f(T^*; F) > 0$ satisfies

$$\int_t^\infty f(T^*; F) dT^* \leq 1 \quad \Rightarrow \quad \int_t^\infty f(T^*; F) e^{-r(T^*-t)} dT^* < 1,$$

where the first (weak) inequality allows for a probability atom at $T^* = \infty$ and the second inequality follows for any $r > 0$. Hence,

$$-1 < \frac{d}{dF} V^*(\hat{\mu}(t), t; F) < 0. \quad (\text{A.5})$$

By (A.1),

$$V^*(\hat{\mu}^*(t; F), t; F) = -F; \quad (\text{A.6})$$

evaluating at the same $\hat{\mu}, t$ point the value function associated to a larger firing cost $F + \Delta$, we can write

$$\begin{aligned} V^*(\hat{\mu}^*(t; F), t; F + \Delta) &= V^*(\hat{\mu}^*(t; F), t; F) + \int_0^\Delta \frac{dV^*(\hat{\mu}(t), t; F)}{dF} dF \\ &\geq V^*(\hat{\mu}^*(t; F), t; F) - \Delta \\ &= -(F + \Delta) \end{aligned}$$

where the inequality follows from (A.5) and the last equality from (A.6). Hence, at all points on the $\hat{\mu}^*(t; F)$ firing boundary the value of continued employment is larger than the value of termination if the firing cost is $F + \Delta$. It follows that the firing boundary $\hat{\mu}^*(t; F)$ is shifted downwards, at all t , by a larger firing cost for given w . Since the $\hat{\mu}(T_i)$ process must pass through $\hat{\mu}^*(t; F)$ before reaching $\hat{\mu}^*(t; F + \Delta)$, tenure length is increased path-by-path by more stringent job security.

Consider next the effect of the (constant) wage rate w on match value. The envelope property of definition (A.3) implies

$$\frac{d}{dw} V^*(\hat{\mu}(t), t; F) = \int_t^\infty \left[\frac{e^{-rT^*} - 1}{r} \right] f(T^*; F) dT^* < 0. \quad (\text{A.7})$$

From (A.4) and (A.7) evaluated at $t=0$, we obtain

$$0 = \frac{dw}{r} \left(1 - E \left[e^{-rT^*} \right] \right) + dF E \left[e^{-rT^*} \right] \quad (\text{A.8})$$

along the locus of firing costs F and wage rates w such that $V^*(\hat{\mu}(0), 0; w, F) = H$. Thus, a higher firing cost F implies a lower willingness to pay on the part of the employer, and a lower wage rate w in equilibrium if the employer under consideration is the one extending the relevant outside offer.

When the firing cost F is imposed on all employers, all wage offers and the equilibrium wage adjust as in (A.8) to keep all outstanding offers' value constant at zero with $H=K$. It can be shown that such wage adjustments yield less frequent turnover. Applying the envelope theorem to $V^*(\hat{\mu}, t; w, F)$,

$$dV^*(\hat{\mu}, t; w, F) = -E_t \left[e^{-r(T^*-t)} \right] dF + E_t \left[e^{-r(T^*-t)} - 1 \right] \frac{dw}{r}.$$

This and (A.8) imply

$$\frac{dV^*(\hat{\mu}, t; w, F)}{dF} = \frac{E[e^{-rT^*}] - E_t[e^{-r(T^*-t)}]}{1 - E[e^{-rT^*}]}. \quad (\text{A.9})$$

To sign the derivative in (A.9), consider that short residual employment spells are assigned much more probability weight when the worker under consideration turns out to have low estimated productivity than at the inception of the match. With a positive discount rate r , then, $E_t[e^{-r(T^*-t)}] < E[e^{-rT^*}]$ in the neighbourhood of the firing boundary. By (A.9), higher firing costs yield higher values of continued employment at those times when termination of the employment relationship is actually being considered. It follows that the firing locus shifts downwards when F moves and w adjusts to keep every match's initial value constant at H , and that turnover takes place less often.