Economics Letters 42 (1993) 43–50

0165-1765/93/$06.00

Turnovers and asymptotic behavior of workers

Leonardo Felli *

The London School of Economics & Political Science, Houghton Street, London WC2A 2AE, UK

Received 22 March 1993
Accepted 14 April 1993

Abstract

In a labor market in which jobs are 'pure search goods' we ask whether workers are successful in their search for the best possible job. We show that workers' search activity ends in a finite amount of time and is successful whenever either changing jobs is costless or its costs are not too high. When turnover costs are high it is possible to envisage situations in which a worker ends up holding for ever a job that is not the best among the ones available to him in the labor market.

1. Introduction

The literature on job search and matching [Mortensen (1978), Jovanovic, (1979a,b)] has addressed a number of issues concerning the average quality of the matches between employers and employees, concerning permanent job separations and wage dynamics and their relationship with turnover, and, more recently [Bertola and Felli (1992), Farber and Gibbons (1992)], concerning the effects of labor market institutions, such as job security or schooling, on turnover and wages.

An issue overlooked by this literature concerns the final outcome of a matching process: the state toward which the search process for a better match converges as time goes by and matches form and dissolves. This is the problem we address in this paper. In particular we ask whether, in a simple model of matching where jobs are 'pure search goods', workers will ever stop searching for a better match and whether this search activity will be successful (in the sense described below). We conclude that the worker's search activity will indeed stop in a finite amount of time. Moreover, if changing job is not costly the search activity will be fully successful: each worker will eventually obtain his best possible job among the available ones. If, on the other hand, moving from one job to an other entails a cost the same success may not be guaranteed. In fact, it may be possible to envisage situations in which a worker ends up forever in a match on which it is possible, in principle, to improve upon in the existing population of employers. Unfortunately, this improvement is overcome, from the worker's view point, by the search costs.

The answers we obtain are somehow less obvious than they may seem at first sight. In

* This is a revised version of a note written while I was a graduate student at the Department of Economics of the Massachusetts Institute of Technology. I wish to thank Giuseppe Bertola, Jane Eberly and Robert Gibbons for very helpful comments. Errors are my own responsibility.

particular, although it is quite intuitive that a search process may eventually converge, it is not at all obvious that such convergence has to occur in a finite amount of time. What this result is ruling out is the possibility for a worker to keep sampling for ever jobs of similar or lower (with respect to his current job) quality. This phenomenon is in principle possible since the population of potential matches is invariant through time. It is the Markov nature of the rule through which job offers are generated which prevents such a phenomenon from occurring in equilibrium. This Markov nature corresponds to the fact that the probability for a worker to get a better job offer and move in the future is independent of time. In a sense, for a worker it is never too late or too early to get a better offer; alternatively, the market does not operate any type of age discrimination. Hence, our result adds one more reason to sustain labor market policies aimed at reducing discrimination toward either young (first employment) or old workers.

The rest of the paper is organized as follows: In Section 2 we introduce a simple model, in Section 3 we analyze the asymptotic behavior of a worker when there are no turnover costs, and in Section 4 when turnover costs are positive. Concluding remarks end the paper.

2. The model

This model considers the problem of permanent job separations – workers' change of employer – when the quality of a match between a worker and an employer is detected by inspection before the beginning of the working activity: jobs are 'pure search goods'. [1]

The following assumptions are essential for the analysis:

Assumption 1. There exists a non-degenerate distribution of workers' productivity (match quality) across different employees. The non-degeneracy of this distribution is due to the assumption that the quality of the match of a single worker differs across prospective employees.

Assumption 2. Employers contract with workers on an individual basis so that the wage paid to a worker reflects the – actual or perceived – success of the match. This assumption allows us to consider the single worker problem.

Assumption 3. The worker stays on his current job until he locates a better match, then a job change occurs. All separations can be interpreted as quits and no layoffs are possible in the model.

Assumption 4. The worker wage is assumed to be equal to his output in the match. In other words, workers have full bargaining power when contracting the wage with employers; alternatively, workers are self-employed and they are searching for their best occupation. Mortensen (1978) proves that if workers are risk-neutral and if discount rates do not differ between workers and firms, this arrangement leads to socially optimal levels of investment and turnover.

Finally, we start our analysis assuming that quits are costless: a worker may change job without incurring any type of cost. We relax this assumption in section 4 and we explore how our results depend on this assumption.

[1] This is clearly a simplifying assumption. However, the main results of the paper do not depend on it. In fact, in the hypothesis that are optimal learning process is required to ascertain the quality of a match and this learning is independent of the worker's job market performance, the process of the optimal stopping times of these learning processes will still satisfy the properties characterized in our analysis.

Under the above assumptions consider the following characterization of a search model.

Each worker does not know his distribution of productivity across employers. This implies that the quality of the match is a random variable, M, from the worker's view point. At the beginning of each period the worker contacts one and only one new employer. Upon contact the quality of the match is immediately ascertained, and the employer makes a wage offer to the worker based on the observed realization of the match quality, $M = m$. Assumption 4 implies that the wage offer is exactly equal to the quality of the match: $w = m$. Everything is as if at the beginning of each period the worker independently draws a wage offer from the distribution of available matches.

We assume that there exist N employers; through Assumption 1 this implies that the random variable M has N realizations:

$$M = m_i\,, \quad \text{where } i \in \{1, \ldots, N\}\,, \tag{1}$$

and the distribution of wage offers is discrete:

$$\Pr\{M(t) = m_i\} = q_i\,, \quad \forall i \in \{1, \ldots, N\}\,. \tag{2}$$

Without any loss in generality we can reorder the realizations of M in the following way:

$$m_i \leq m_j\,, \quad \forall i \geq j\,. \tag{3}$$

Finally, we assume that recall options are available. In other words the worker may draw the same wage offer more than one time. This assumption guarantees that the distribution of potential offers is stable over time.

In the first period, $t = 1$, the probability of having a match with employer j is

$$\Pr\{E(1) = j\} = \Pr\{M(1) = m_j\} = q_j\,, \tag{4}$$

where $[E(t) = j]$ denotes the event: 'the worker is employed in firm j in period t'.

In the second period, $t = 2$, we have

$$\Pr\{E(2) = j\} = \sum_{i=1}^{N} \Pr\{E(2) = j \mid E(1) = i\} q_i\,. \tag{5}$$

In general,

$$\Pr\{E(t) = j\} = \sum_{i=1}^{N} \Pr\{E(t) = j \mid E(t-1) = i\}\ q_i\,. \tag{6}$$

It is now possible to ascertain that

$$\Pr\{E(t) = j \mid [E(t-1) = i] \cap H(t-2)\} = \Pr\{E(t) = j \mid E(t-1) = i\}\,, \tag{7}$$

where $H(t-2)$ is a history, i.e. a set of realizations for the random variables $E(t-2), \ldots, E(1)$. In other words the process $\{E(t)\}$ satisfies the first-order Markov property [Feller (1970, ch. XV)].

Let us consider the transition probability:

$$p_{ij} \equiv \Pr\{E(t) = j \mid E(t-1) = i\}\,. \tag{8}$$

The value of this probability is fully determined by the worker's optimal decision whether to change his current employer or not. When turnover is costless such an optimal decision corresponds to the following rule. A worker will move if $m_j > m_i$ and he will stay with his current employer if $m_j \le m_i$. This rule is optimal in a frictionless world like the one we are describing. In fact, for a worker in this world there exists no reason whatsoever to condition his decision to move either on his past history or on his future perspectives. Hence, the transition probability defined in (8) is equal to zero if $j > i$; is equal to $\sum_{h=j}^{N} q_h$ if $j = i$; and is equal to q_j if $j < i$. This transition probability is clearly independent of t and satisfies the first-order Markov property. Thus, the process $\{E(t)\}$ of the worker's job changes is a *finite Markov chain*, whose matrix of transition probabilities is defined in the following way:

$$p_{ij} = \begin{cases} 0, & \text{if } i < j, \\ \sum_{h=j}^{N} q_h, & \text{if } i = j, \\ q_j, & \text{if } i > j. \end{cases} \tag{9}$$

Denote $\boldsymbol{P}$ the matrix of transition probabilities, and

$$\boldsymbol{q} \equiv (q_1, \ldots, q_N), \tag{10}$$

$$\boldsymbol{e}(t) \equiv [e_1(t), \ldots, e_N(t)], \tag{11}$$

where $e_h(t) = \Pr\{E(t) = h\}$. We have

$$\boldsymbol{e}(t+1) = \boldsymbol{q} \cdot \boldsymbol{P}^t, \tag{12}$$

where the dot indicates the usual (rows by columns) matrix multiplication and $\boldsymbol{P}^t$ is the tth power of the matrix $\boldsymbol{P}$ ($\boldsymbol{P}^0$ being the identity matrix).

3. The asymptotic behavior of workers: Costless turnover

We are now in the condition to address the main concern of this paper: the problem of the existence of a stopping point T in the worker's job changing process and the optimality of the worker's decision not to change job any more. In this section we address this issue under the hypothesis of costless turnover. From a technical view point we are asking whether the Markov chain defined by (9), (10) and (11) has an ergodic absorbing state, whether the process converges to such a state, and whether this state represents the optimal outcome for the worker: $M(T) = m_1$.

The matrix $\boldsymbol{P}$ is diagonal with the element $p_{11} = 1$ and positive elements – not greater than one – on the lower left of the principal diagonal. Without loss of generality we can assume that $m_1 > m_2$, which implies that p_{11} is the only unitary element of $\boldsymbol{P}$. The matrix of transition probabilities is then written in its *canonical form* and there does exist one ergodic class which consists of only one absorbing state. [2] On the other hand, all other states are transient and N is a null state [Feller (1970, ch. XV)].

In economic terms this means that the worker will never accept the wage offer of firm N, unless he is searching for his first employment. If the worker is in firm i he will always change job if he happens to have a wage offer by firm j, $i > j$. Finally, the worker will never change job if he reaches firm 1.

We can now apply the ergodic theory of Markov chains [Feller (1970, ch. XV)] to study the behavior of the worker when t increases. The following results hold.

[2] If $m_1 = m_2 > m_3$, there exist two ergodic classes; each one includes one state only and states 1 and 2 are both absorbing.

(i) The worker will certainly get the *optimal offer*, m_1, whatever the employer with which he had the first match. This is because in a finite Markov chain the probability of the system staying forever in the transient states is zero. [3]

(ii) The number of job changes necessary to reach the worker's best match is *finite*. This is because the average waiting time for the system to reach the worker's best match – the absorbing state – is finite. [4] In other words, there exists a finite stopping time T in which the worker will reach his optimal match: $E(T) = 1$.

(iii) The increasing experience of the worker – number of jobs changes – progressively reduces his uncertainty about the quality of the possible matches with the N employers. In fact, the distribution $e(t)$ as t increases converges to a degenerate distribution which concentrates the unitary mass on the realization $E(T) = 1$, and the process $\{E(t)\}$ converges to a degenerate stationary process. [5]

4. The asymptotic behavior of workers: Costly turnover

In this section we relax the assumption of costless turnover and explore how this modifies the results presented in section 3. Denote by h the turnover cost each worker has to incur whenever he decides to change job. The simplest way to interpret such a cost is in terms of relocation or moving expenses which the worker has to incur to change employer. For notational simplicity, we take h to be the same for every job and to be known to the worker before the realization of a match. Note that there is no loss of generality in assuming that the worker, as opposed to the employer, will incur the cost. In fact, this is the only possible equilibrium allocation of costs in a model in which workers are basically self-employed. As a matter of fact, h may be re-interpreted as a specific investment in the new activity the worker may decide to undertake. Since in our model workers are the residual claimant of all the surplus produced by a match, they are the only agents with enough incentives to undertake such specific investments and pay h. [6]

In this case the characterization of the stochastic process underlying each worker's job change needs to be modified. In particular, the transition probability defined in (8) will be fully determined by the worker's new optimal decision rule which takes into account the positive turnover cost, h. In fact, a fully rational, forward-looking worker will consider a job offer 'better' than his current one if it entails expected present discounted benefits which are high enough to cover the turnover cost. In formal terms, denoting δ as the worker's discount rate, in period t a worker, whose current wage is m_i, will decide to accept a job offer m_j if and only if the following condition is satisfied:

$$\mathrm{Exp}\left\{\sum_{\mathcal{T}=t}^{\mathcal{T}} (m_j - m_i)\delta^{\mathcal{T}-t}\right\} \geq h\,, \tag{13}$$

where $\mathrm{Exp}\{\cdot\}$ denotes the expectation operator, and $\mathcal{T}$ the random stopping time in which the worker will eventually decide to quit employer j because of the arrival of a new 'better' offer.

[3] Feller (1970, p. 401).
[4] Feller (1970, p. 403).
[5] This can be proved by solving the system of linear equations $u = u \cdot P$, where u is a vector of N unknowns.
[6] Employers will find it optimal to pay the turnover cost h only when the distribution of the match surplus between employers and employees is non-degenerate [Bertola and Felli (1992)].

Note that the expectation (13) is taken on the stopping time $\mathcal{T}$. The inequality (13) may be re-written as

$$(m_j - m_i)\left(\frac{1-\delta\bar{\delta}_j}{1-\delta}\right) \geq h\,, \tag{14}$$

where we denote $\bar{\delta}_j \equiv \mathrm{Exp}\{\delta^{(\mathcal{T}-t)}\}$ as the expectation of the random variable $\delta^{(\mathcal{T}-t)}$ which takes a value for every possible realization of the random stopping time $\mathcal{T}$ with an infinite countable support

$$\{t, t+1, \ldots, t+n, \ldots\}\,.$$

The analytical computation of $\bar{\delta}_j$ is a formidable task since the probability of any realization of $\mathcal{T}$ is the probability that in any future period a condition equivalent to (14) will be verified and such a condition will itself depend on $\bar{\delta}_j$. Furthermore, such probability will depend on the offer m_j the worker is evaluating, since only offers which are at least greater than m_j will induce a future change of job; from here the dependence of $\bar{\delta}_j$ upon j. Nevertheless, we have enough information on $\bar{\delta}_j$ to be able to characterize the worker's asymptotic behavior. In particular, all we need is that $\bar{\delta}_j < 1$, for every $j = 1, \ldots, N$, which is implied by the definition of $\bar{\delta}_j$ and $\bar{\delta}_1 = 0$ which can be proved recalling that m_1 is the highest possible offer for any worker; hence if such an offer were accepted, no job change will ever occur again: $\mathcal{T} = \infty$.

We can now proceed to evaluate the transition probability, p_{ij}, defined in (8), when turnover is costly: $h > 0$. Given a current employer i, define $\mathcal{J}_i$ as the set of employers' indexes for which condition (14) is satisfied:

$$\mathcal{J}_i \equiv \left\{ j \mid j \in \{1, \ldots, N\} \text{ and } (m_j - m_i)\left(\frac{1-\delta\bar{\delta}_j}{1-\delta}\right) \geq h \right\}.$$

The matrix $\boldsymbol{P}$ of transition probabilities p_{ij} takes the following values:

$$p_{ij} = \begin{cases} 0\,, & \text{if } j \not\in \mathcal{J}_i \text{ and } j \neq i\,, \\ \sum_{h \not\in \mathcal{J}_i} q_h\,, & \text{if } j = i\,, \\ q_j\,, & \text{if } j \in \mathcal{J}_{ij}\,. \end{cases} \tag{15}$$

Observe, first, that p_{ij}, as re-defined in (15), is once again independent of t and satisfies the first-order Markov property. Hence, the new process $\{E(t)\}$ of the worker's job changes is still a *finite Markov chain*. This means that it is still possible to use the ergodic theory of Markov chains to study the behavior of a worker when t increases. The implication of this analysis, though, will be different from the ones derived in section 3 depending on the size of the cost h which determines the values of the transition probabilities in (15).

We shall explore the robustness of the results presented in section 3 to changes in the values of the turnover cost h starting from the simplest case: a very small h. In particular, whenever

$$h \leq \min_i \left\{ (m_i - m_{i-1})\left(\frac{1-\delta\bar{\delta}_i}{1-\delta}\right) \right\}, \tag{16}$$

the values of the transition probabilities (15) coincide with the values (9). In fact, in such a case (16) implies that condition (14) is satisfied whenever $m_j > m_i$. Hence, the characterization of the worker asymptotic behavior, presented in Section 3, will still apply.

The situation is, instead, more interesting in the case in which (16) does not hold. In such a case the conclusions of section 3 need to be substantially altered. In particular, it is still true that the matrix $\boldsymbol{P}$, as defined in (15), is still diagonal with the element $p_{11} = 1$ and positive elements – not greater than one – on the lower left of the principal diagonal; but nothing guarantees that p_{11} is the only unitary element of the matrix. In particular, consider the case in which

$$h > (m_1 - m_2)\left(\frac{1}{1-\delta}\right). \tag{17}$$

In such a case, even if $m_2 < m_1$, the probability of staying in state 2 is one as well: $p_{22} = 1$. This implies that the matrix of transition probability, in its *canonical form*, shows the existence of at least two ergodic classes, both consisting of only one state: state 1 and state 2. In economic terms, this implies that any worker than ends up matched with employer 2 will hold that job forever, since by (17) the expected benefits from a movement to the match with employer 1 are not enough to cover the turnover cost, h. Thus, in the case in which (17) holds, we can conclude:

(i′) The worker will not necessarily get the *optimal offer*, m_1. He may as well end up getting offer m_2 without ever moving from that job. Which offer the worker will get as t gets large depends on the match he draws in the first period in the labor market. However, the probability of the system staying forever in the transient states is still zero.

(ii′) The number of job changes necessary to reach the worker's 'final' match – the one from which he will never move – is still *finite*. This is because the average waiting time for the system to reach one of the final matches – the absorbing states – is finite. Once again there exists a finite stopping time T (or T') in which the worker will reach his final match: $E(T) = 1$, $E(T') = 2$.[7]

(iii′) The increasing experience of the worker – number of job changes – progressively reduces his uncertainty about the quality of the possible matches with the N employers. In fact, the distribution $e(t)$ as t increases, converges to a degenerate distribution which concentrates the unitary mass on one of the realization, $E(T) = 1$ or $E(T') = 2$, and the process $\{E(t)\}$ converges to a degenerate stationary process. This result implies that the worker may end up not holding his best match and being perfectly aware of it since when he had the chance to change to the best match he refused because of the high turnover costs.

Finally, observe that it is possible to envisage a situation in which turnover costs are high enough to prevent any form of turnover in the model. In fact, consider the case in which the following condition holds:

$$h > (m_1 - m_N)\left(\frac{1}{1-\delta}\right). \tag{18}$$

In such a situation turnover costs are so high that every worker will stay forever with his first match. In formal terms, $p_{ii} = 1$ for every $i \in \{1, \ldots, N\}$; the matrix of transition probabilities coincides with the identity matrix and hence every state is absorbing.

5. Summary and conclusions

In this paper we have addressed the problem of the existence and optimality of a final stopping point in a worker's job search.

Wherever a worker knows the distribution of available jobs, if the job changing mechanism

[7] Note that nothing guarantees that the finite stopping time will coincide, whether the process converges to state 1 or 2.

satisfies a Markov property and turnover is costless – or turnover costs are not too high – both objectives can be reached. In fact, the worker reaches a stationary situation – he does not change job whatever his new offer is – in a finite number of changes and this stationary situation yields the optimal match which is reflected in the highest wage offer. If, on the other hand, turnover costs are high, the worker will still reach in a finite time the situation in which he will not change job whatever offer he receives. But he may end up holding for ever a job that is not the optimal match.

Two final observations may be added.

The first concerns the conditions under which these results generalize. The Markov property of the job changing process is essential to this purpose. This does not mean that the process has to take the form of a Markov chain; in this paper we used this simple form because of its easily interpretable asymptotic behavior. As a matter of fact, the Wiener process of Jovanovic (1979a) is a Markov process itself. Therefore more complex and realistic ways of modeling the job searching process may be envisaged. The essential aspect of the Markov property is the fact that the probability of receiving in the future a better offer is independent of the time spent by the worker in the labor market. As we argued in the introduction, this suggests an additional reason for policy interventions aimed at reducing the age discrimination in the labor market.

Furthermore, the comparison between the final allocation of workers, when turnover costs are essentially zero and when they are strictly positive, makes room for policies aimed at reducing turnover costs and increasing workers' mobility. Moreover, our analysis sheds light on one of the long-run inefficiencies introduced in the economic system by high mobility costs.

References

Bertola, Giuseppe and Leonardo Felli, 1992, Job matching and the distribution of producer surplus, mimeo, Princeton University and London School of Economics.

Farber, Henry and Robert Gibbons, 1992, Learning and wage determination, mimeo, Princeton University and Cornell University.

Feller, William, 1970, An introduction to probability and its applications, 3rd edn., Vol. 1 (Wiley, New York).

Jovanovic, Boyan, 1979a, Job matching and turnover, Journal of Political Economy 87, 972–990.

Jovanovic, Boyan, 1979b, Firm-specific capital and turnover, Journal of Political Economy 87, 1246–1260.

Mortensen, Dale, 1978, Specific human capital and labor turnover, Bell Journal of Economics 9, 572–586.